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Verification of Timed Automata by CEGAR-Based Algorithms

Scientific Students' Associations Report

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Abstract Nowadays safety-critical systems are becoming increasingly prevalent, however, faults in their behaviour can lead to serious damage. Therefore, it is extremely important to use mathematically precise verification techniques during their development. One of them is formal verification, that is able to find design problems from early phases of the development. However, the complexity of safety-critical systems often prevents successful verification. This is particularly true for real-time systems: the set of possible states and transitions can be large or infinite, even for small timed systems. Thus, selecting appropriate modelling formalisms and efficient verification algorithms is very important. One of the most common formalisms for describing timed systems is the timed automaton that extends finite automata with clock variables to represent the elapse of time.

When applying formal verification, reachability becomes an important aspect – that is, examining whether the system can reach a given erroneous state during its execution. The complexity of the problem is exponential even for simple timed automata (without discrete variables), thus it can rarely be solved for large models. A possible solution to overcome this deficiency is to use abstraction, which simplifies the problem to be solved by focusing on the relevant information. However, the main difficulty when applying abstraction-based techniques is finding the appropriate precision, which is coarse enough to reduce complexity but fine enough to be able to solve the problem. Counterexample-guided abstraction refinement (CEGAR) is an iterative method starting from a coarse abstraction and refining it until a sufficient precision is reached.

The goal of my work is to develop efficient algorithms for the verification of timed automata. In my work I examine, and develop CEGAR-based reachability algorithms applied to timed automata and I integrate them to a common framework where components of different algorithms are combined to form new and efficient verification methods. The framework offers two realizations of the CEGAR approach: one of them applies abstraction to the automaton in order to gain an overapproximation of the set of reachable states (the state space), and the other applies abstraction directly to the state space.

I demonstrate the efficiency of the developed algorithms by measurements on examples that are commonly used to benchmark model checking algorithms for timed automata.

Kivonat A napjainkban egyre inkább elterjedő biztonságkritikus rendszerek hibás működése súlyos károkat okozhat, emiatt kiemelkedően fontos a matematikailag precíz ellenőrzési módszerek alkalmazása a fejlesztési folyamat során. Ennek egyik eszköze a formális verifikáció, amely már a fejlesztés korai fázisaiban képes felfedezni tervezési hibákat. A biztonságkritikus rendszerek komplexitása azonban gyakran megakadályozza a sikeres ellenőrzést, ami különösen igaz az időzített rendszerekre : a lehetséges állapotok és átmenetek halmaza (állapottér) akár kisméretű rendszerek esetén is hatalmas, vagy akár végtelen nagy is lehet. Ezért kiemelkedően fontos a megfelelő modellezőeszköz valamint hatékony verifikációs algoritmusok kiválasztása. Az egyik legelterjedtebb formalizmus időzített rendszerek leírására az időzített automata, ami a véges automata formalizmust óraváltozókkal egészíti ki, lehetővé téve az idő múlásának reprezentálását a modellben.

A formális verifikáció egyik alapvető feladata az állapotelérhetőségi analízis, amely során azt vizsgáljuk, hogy lehetséges-e, hogy a rendszer működése során elér egy adott hibaállapotba. A probléma komplexitása már egyszerű (diszkrét változó nélküli) időzített automaták esetén is exponenciális, így nagyméretű modellekre ritkán megoldható. Ezen probléma leküzdésére nyújt egy lehetséges megoldást az absztrakció módszere, amely a releváns információra koncentrálva próbál meg egyszerűsíteni a megoldandó problémán. Az absztrakció-alapú technikák esetén azonban a fő probléma a megfelelő pontosság megtalálása. Az ellenpélda vezérelt absztrakciófinomítás (counterexample-guided abstraction refinement, CEGAR) iteratív módszer, amely a rendszer komplexitásának kézben tartása érdekében egy durva absztrakcióból indul ki és ezt finomítja a kellő pontosság eléréséig.

Munkám célja hatékony algoritmusok fejlesztése időzített rendszerek verifikációjára. Munkám során időzített automatákra alkalmazott CEGAR-alapú elérhetőségi algoritmusokat vizsgálok, fejlesztek és közös keretrendszerbe foglalom őket, ahol az algoritmusok komponensei egymással kombinálva új, hatékony ellenőrzési módszerekké állnak össze. A keretrendszer a CEGAR módszer kétféle megvalósítását is lehetővé teszi : az egyik az elérhető állapotok halmazának (az állapottérnek) felülbecsléséhez az automatát egyszerűsíti, míg a másik közvetlenül az állapottéren alkalmaz absztrakciót.

A kifejlesztett algoritmusok hatékonyságát méréseken keresztül demonstrálom, amelyek bemeneteit időzített automáták modellellenőrzésére kifejlesztett algoritmusok összehasonlító elemzéséhez gyakran használt modellek közül választottam.

Chapter 1

Introduction

Safety critical systems, where failures can result in serious damage, e.g. death, are becoming more and more ubiquitous. Consequently, the importance of using mathematically precise verification techniques during their development is increasing.

Formal verification techniques are able to find design problems from early phases of the development, however, the complexity of safety-critical systems often prevents their successful application. The behaviour of a system is described by the set of states that are reachable during execution (the state space) and formal verification techniques like model checking examine correctness by exploring it explicitly or implicitly. However, the state space can be large or infinite, even for small instances. Thus, selecting appropriate modelling formalisms and efficient verification algorithms is very important. One of the most common formalisms for describing timed systems is the formalism of timed automata that extends finite automata with clock variables to represent the elapse of time.

When applying formal verification, reachability becomes an important aspect – that is, examining whether a given erroneous state is reachable from an initial state. The complexity of the problem is exponential even for simple timed automata (without discrete variables), thus it can rarely be solved for large models. A possible solution to overcome this deficiency is to use abstraction, which simplifies the problem to be solved by focusing on the relevant information. However, the main difficulty when applying abstraction-based techniques is finding the appropriate precision: if an abstraction is too coarse it may not provide enough information to prove the desired property, whereas if an abstraction is too fine it may cause complexity problems. Counterexample-guided abstraction refinement (CEGAR) is an iterative method starting from a coarse abstraction and refining it until a sufficient precision is reached.

CEGAR [6] has been successfully applied to many modelling formalisms, such as Markov Decision Processes [19], Hybrid Automata [22] and Petri Nets [14]. The goal of my work is to develop efficient CEGAR-based algorithms for the verification of timed automata. There are several existing approaches in the literature for CEGAR-based verification of timed automata, including [17] where the abstraction is applied on the locations of the automaton, [20] where the abstraction of a timed automaton is an untimed automaton and [11, 15], and [21] where abstraction is applied on the clock variables of the automaton.

In my work I examine various CEGAR-based reachability algorithms applied to timed automata and I integrate them to a common framework where components of different algorithms are combined to form new and efficient verification methods. Many of the implemented techniques are known from the literature, but most of them were invented for other formalisms, and I had to adapt them to timed automata. Other implemented techniques are my own contribution. The developed framework offers two realizations of the CEGAR approach: one of them applies abstraction to the automaton in order to gain an overapproximation of the set of reachable states (most known algorithms are based on this approach), and the other applies abstraction directly to the state space.

The correctness and the efficiency of the created algorithms are demonstrated by measurements. The inputs of the measurements are chosen from a set of example timed automata that are widely used to compare model checking algorithms.

The paper is organized as follows. Chapter 2 provides basic knowledge about mathematical logic, formal verification and timed automata, Chapter 3 explains the implemented algorithms and how they can be combined in the developed framework, and Chapter 4 describes the implementation environment, and summarizes the results of the measurements. Finally, Chapter 5 concludes my work.

Chapter 2

Background

2.1 Mathematical logic

Mathematical logic is useful for deciding correctness of systems. This section provides some insight about propositional logic, first order logic, and the satisfiability problem. Difference logic is also introduced.

2.1.1 Propositional logic

Propositional logic is concerned with the study of formulae of boolean variables, and deciding whether they are true or false under a given assignment. Propositional formulae are composed of *truth symbols* $\top$ (true) and $\bot$ (false), and propositional *variables* $p, q, \ldots$ with the use of *logical connectives*. A formula φ can be an *atom* (a truth symbol or a variable) or can be constructed from other logical formulae with the following connectives:

- negation: $\neg\varphi$ is evaluated true iff φ is evaluated false (formal equivalent of 'not'),
- conjunction: $\varphi_1 \wedge \varphi_2$ is evaluated true iff both φ_1 and φ_2 are evaluated true (formal equivalent of 'and'),
- disjunction: $\varphi_1 \vee \varphi_2$ is evaluated true iff at least one of φ_1 and φ_2 is evaluated true (formal equivalent of 'or'),
- implication: $\varphi_1 \rightarrow \varphi_2$ is evaluated true iff φ_1 is evaluated false or both φ_1 and φ_2 are evaluated true (formal equivalent of 'if ... then'),
- equivalence: $\varphi_1 \leftrightarrow \varphi_2$ is evaluated true iff both φ_1 and φ_2 are evaluated true or both φ_1 and φ_2 are evaluated false (formal equivalent of 'if and only if').

Note: disjunction, implication, and equivalence can be expressed using negation and conjunction. These operators are only defined to simplify usage.

The (boolean) satisfiability problem (SAT, for short) can be defined as follows [3].

Input : A propositional logic formula φ.

Output : *Yes* if φ is satisfiable (i.e. it is possible to ground the variables appearing in φ to truth symbols so that φ is evaluated true), *No* otherwise.

SAT is NP-complete [9] but in practice there are efficient solvers [16].

2.1.2 First order logic

Propositional logic is useful, however, sometimes its expressive power is not enough. First order logic is a extends propositional logic with predicate symbols, function symbols and quantifiers $\exists$ (existential quantifier) and $\forall$ (universal quantifier).

The basic elements of first order logic are *terms*. Variables and constant symbols (0-ary function symbols) are terms, as well as n-ary functions applied to n terms. In first order logic an atom can be $\top, \bot$ or an a n-ary predicate symbol applied to n terms. Formulae are constructed by applying connectives (the same as in case of propositional logic) and quantifiers to atoms.

The satisfiability problem can be extended to first order logic formulae, but it is undecidable [5, 23]. However, there is a variant of the problem that is applicable, and solvable for most practical problems. The key idea is to formalize structures.

Definition 2.1 A *first order theory* $\mathcal{T}$ is a pair $(\Sigma, \mathcal{A})$ [4] where

- Σ is the *signature*, i.e. the set of constant, function and predicate symbols and
- $\mathcal{A}$ is the set of *axioms* where an axiom is a first order logic formula that has no quantifiers in it, and Σ contains all constants, functions and predicates appearing in it.

The *Satisfiability Modulo Theories* problem (SMT for short) can be defined as follows.

Input : A theory $\mathcal{T} = (\Sigma, \mathcal{A})$, and Σ-formula φ.

Output : *Yes* if φ is satisfiable in $\mathcal{T}$, *No* otherwise.

In many practical theories, SMT becomes decidable [18].

2.1.3 Difference logic

An atom in (integer) difference logic is a logical expression of the form $x - y < n$ or $x - y \leq n$ where x and y are variables defined over $\mathbb{Z}$ and n is a constant. A difference logic formula φ is a conjunction of one ore more atoms. In case of difference logic, satisfiability is not only decidable, but decidable in polynomial time [18].

Since the framework presented in this paper relies on a SAT (SMT) solver (as it is explained in Chapter 4), this paper does not address the algorithms for deciding satisfiability (if it is decidable). For more information on satisfiability, the reader is referred to [4].

2.2 Formal verification

Formal verification is the act of proving the correctness or incorrectness of a system in a mathematically precise way. Model checking is an automatic formal verification technique based on systematic state space traversal of system, where the system has to be represented by a formal model, and the requirements of the system has to be a property of the system formally defined as logical formulae [8]. Verification can be performed by proving that the system (represented by the formal model) satisfies that property.

2.2.1 Modelling formalisms

It is important to find the appropriate representation of the model i.e. the appropriate formalism in order to be able to model and check the property. This paper focuses on *behavior properties* examined on *behavioral models*.

In formal verification behaviour is often represented with state-based formalisms: the system's behaviour is partitioned to a (not necessarily finite) set of *states* (the *state space*), that is complete and excluding – i.e. at any given time the system is in exactly one of the states. The state of the system determines the possible behaviours. Because of this, it is important to choose the granularity of the state space in a way that it can be decided what behaviour is possible and what is not, solely based on the system's current state.

Finite automata

The formalism of finite automata is one of the most common formalisms for modelling behaviour. This way the system is described by a finite set of possible states, and a set of steps defining the system's state changes.

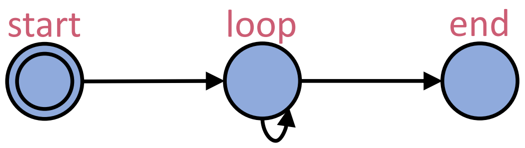


Figure 2.1 Example of a finite automaton

Definition 2.2 Formally a *finite automaton*, or state machine $\mathcal{A}$ is a tuple $\langle S, s_0, T \rangle$ where

- S is a finite set of states,
- $s_0 \in S$ is the initial state and
- $T \subseteq (S \times S)$ is a set of transitions.

Structurally $\mathcal{A}$ can be represented as a directed graph $G_{\mathcal{A}}$ where $V(G_{\mathcal{A}}) = S$ and $E(G_{\mathcal{A}}) = T$. The system's operation is described as follows.

Initially, the system is in s_0. The system can change its state to some other state s_1 iff $(s_0, s_1) \in T$. from s_1 it can change its state to s_2 iff $(s_1, s_2) \in T$, and so on.

Example 2.1 Let $\mathcal{A}$ be a finite automaton with states *start*, *loop*, and *end*, and transitions *start* $\to$ *loop*, *loop* $\to$ *loop*, and *loop* $\to$ *end*, as depicted in Figure 2.1. The initial state is *start*, denoted by double outline.

Operation starts form the initial state, *start*. The system can step to *loop* where it can step to *end* or *loop* (itself). Since it is always a possibility to stay in the current state, loop transitions don't have importance in finite automata.

This formalism is easy to use and verify, but its expressive power is not sufficient: there are many types of behaviours that can't be modelled this way.

Finite automaton extended with variables

Many extensions of the finite automaton are known with various levels of expressive power. The following extension lifts the level of expressive power to that of Turing-machines: extending the automaton with variables.

Definition 2.3 A finite automaton extended by variables could be (briefly) defined as a tuple $\langle L, l_0, v_0, E, I \rangle$ operating on a set of variables $\mathcal{V}$ where

- L is the set of control locations,

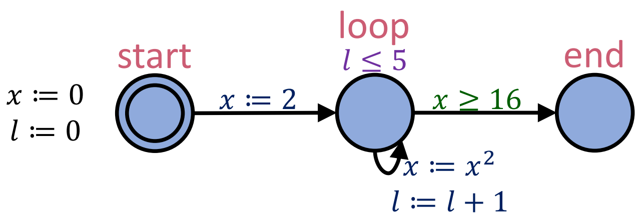


Figure 2.2 Extended finite automaton

- $l_0 \in L$ is the initial location,
- v_0 is a function assigning an initial value to each variable $x \in \mathcal{V}$,
- $E \subseteq L \times \mathcal{B} \times \mathcal{U} \times L$ is the set of edges (where $\mathcal{B}$ can be briefly described as the set of bool valued first order logic formulae constructed from the variables of $\mathcal{V}$, and $\mathcal{U}$ can briefly defined as the set of unary functions operating on subsets of $\mathcal{V}$ assigning new values to variables) and
- $I : L \to \mathcal{B}$ is a function assigning invariants to locations.

Since these automata's behaviour depends on the current values of the variables (the current *valuation*), the basic parts of the model can not be called states. Instead, they are called control locations. For similar reasons, the edges of the graph are now called edges in the formalism as well, and they are more expressive: an edge $e = (l, g, a, l')$ represents a transition form l to l', with a guard g and an assignment function a. A guard is a condition that has to be satisfied in order for the transition to be enabled. The function a describes how the values of some variables are changed during the transition. Locations can have invariants, that are conditions that have to be satisfied while the system stays in that location.

The system's operation starts from the control location l_0, and the variables are initialized as v_0 defines. A system can transition from l to some l' if there exist an edge $e = (l, g, a, l')$ where g is satisfied by the variables current values, and $I(l')$ is satisfied by the values a assigns to the variables (or their current value if a is undefined on them).

Example 2.2 Let $\mathcal{A}$ be an extended version of the automaton in Figure 2.1 as depicted in Figure 2.2. The introduced variables are x and l. The variable l is used as a loop counter. The edge *start* → *loop* assigns 2 to x, *loop* → *loop* squares x and increases the value of the loop counter, *loop* → *end* doesn't affect the values of the variables, but it is only enabled when x is a least 16. Location *loop* has an invariant ensuring that the loop edge is taken at most 5 times.

Operation starts in state $\langle start, x = 0, l = 0\rangle$. When the system changes its state it can only take edge *start* → *loop*, resulting in state $\langle loop x = 2, l = 0\rangle$. Edge *loop* → *end* is not enabled, but the system can take the loop edge resulting in state $\langle loop x = 4, l = 1\rangle$. The edge to *end* is still not enabled, but after taking the loop edge again ($\langle loop x = 16, l = 2\rangle$) it becomes enabled. The system may keep taking the loop edge up to three times (after which the invariant prohibits it, because it would increase l), or transition to *end*.

2.2.2 Reachability analysis

During formal verification, one of the most important questions is reachability: deciding whether a system can step into a given state. In many cases, the state in question represents an erroneous state and the desired outcome of model checking is that it is unreachable. The problem can be defined as follows.

Input : A system S and a state s_{err}.

Output : *Yes* if it is possible for S to operate in a way that it eventually steps in s_{err}, *No* otherwise.

When the answer is *Yes*, it is useful to provide a *counterexample*: an execution trace $\sigma = s_0 \rightarrow s_1 \rightarrow \cdots \rightarrow s_{err}$ setting the system's state to s_{err} where s_i-s are states and $s_i \rightarrow s_{i+1}$ notations represent possible transitions of the system from s_i to s_{i+1}.

Each formalism has its own interpretation of the problem – regarding how erroneous states and execution traces are described. For example, in case of finite automata, the problem can be interpreted as follows.

Input : A finite automaton $\mathcal{A} = \langle S, s_0, T\rangle$ and a state $s_{err} \in S$.

Output : A sequence of states and transitions $\sigma = s_0 \xrightarrow{t_0} s_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} s_{n+1} = s_{err}$ ($s_i \in S, t_i = (s_i, s_{i+1}) \in T$ for all $0 \le i \le n$) if the $\mathcal{A}$ can reach s_{err} , *No* otherwise.

This problem can be solved by any pathfinding algorithm (e.g. breadth first search or depth first search) executed on $G(\mathcal{A})$, where $G(\mathcal{A})$ denotes the graph representation of a finite automaton $\mathcal{A}$.

Example 2.3 Let the input be the previous automaton in Figure 2.1 and it's state *end*. A pathfinding algorithm finds a path, e.g. *start* → *loop* → *end*, proving the state is reachable.

In case of extended finite automata the problem can be interpreted as follows.

Input : A finite automaton $\mathcal{A} = \langle L, l_0, v_0, E, I\rangle$ and a control location $l_{err} \in L$.

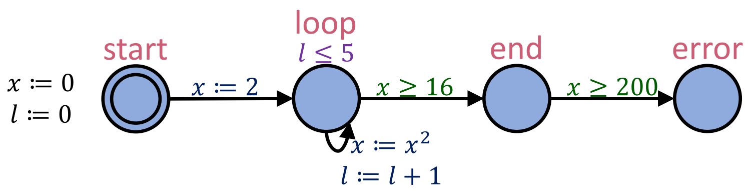


Figure 2.3 Automaton for checking reachability

Output : A sequence of locations and enabled transitions $\sigma = l_0 \xrightarrow{e_0} l_1 \xrightarrow{e_1} \cdots \xrightarrow{e_n} l_{n+1} = l_{err}$ if the S can reach s_{err}, *No* otherwise.

Note, that in this case error states are defined solely by the location, however, it is easy to reduce a problem where the values of the variables are also constrained into this form.

Example 2.4 Consider the previous system $\mathcal{A}$ depicted in Figure 2.2 with the erroneous states defined as the set of states where the current location is *end* and $x \geq 200$.

A new automaton can be constructed by extending $\mathcal{A}$'s set of location with a new location *error* that is only reachable if $\mathcal{A}$ can reach location *end* with $x \geq 200$. The new automaton is depicted in Figure 2.3.

The described problem is unsolvable as well as the reachability problem for all other Turing-complete formalisms. This is one of the reasons why the modelling formalism has to be chosen carefully. A simple formalism may not have enough expressive power to precisely model the system, while verification of more complex formalisms may be ineffective or even impossible.

State space exploration

Even if reachability is undecidable (or just inefficient), there are many methods and approaches on how to gain useful information on the problem. The most obvious approach is *state space exploration*.

Definition 2.4 The *state space* of a system is the set of states that are reachable from the initial state by a sequence of enabled transitions.

The idea of state space exploration is to systematically enumerate all possible states in the state space. If the erroneous state is found, the system is proven to be incorrect. Otherwise if all possible states are listed (in case of a finite state space) and no erroneous

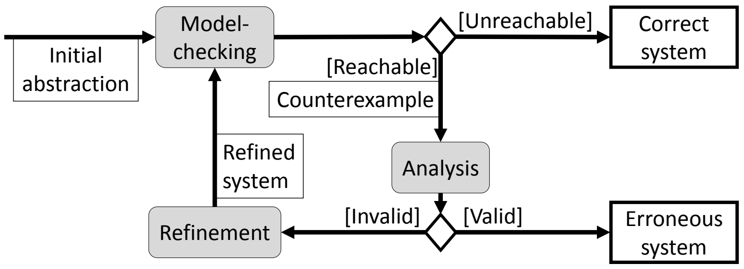


Figure 2.4 Counterexample-guided abstraction refinement

state is found, the system is proven to be correct. (In case of an infinite state space this naive procedure will never terminate.)

One of the simplest ways to explore the state space is to construct a search-tree. The root of the tree is the initial state s_0 of the system. The state space is explored by iteratively choosing a leaf with a state s of the tree, and introducing a new edge for all possible enabled transitions (s, s') pointing to a new node with a state s'. This way, all possible execution traces are explored, but states may appear more than once if there are more execution traces to reach them.

It is also possible to construct a state graph, where the state space is explored similarly, except one state can only appear once. This helps reducing the size of the graph, however (in case of an infinite state space) it still might be infinite. Infinite state spaces can never be completely explored this way (by explicit exploration), however, sometimes, when small counterexamples are expected, it is not necessary.

Consider the *bounded reachability problem*.

Input : A system S, a state s_{err}, and a bound k.

Output : A counterexample, if S can reach s_{err} in at most k transitions, *No* otherwise.

This problem is decidable, even for finite automata with variables. Even so, explicit state space exploration can not be considered an efficient method.

2.2.3 CEGAR

One of the possible approaches to perform model checking more efficiently is to use abstraction [7]. A less detailed system model is constructed that hides unimportant parts of the behavior providing a model of complex state space overapproximating the original one. The idea of counterexample guided abstraction refinement (CEGAR) [6] is to apply model checking to this simpler model, and then examine the results on the original one. The idea is illustrated in Figure 2.4.

First, an abstract system is constructed. The key property of abstraction is that the state space of the abstract system overapproximates that of the original one, but it is less complex and thus model checking can be performed more efficiently on the abstract system.

Model checking is performed on the abstract model. If the target state is unreachable in the abstract model, it is unreachable in the original model as well. Otherwise the model checker produces an abstract counterexample – an execution trace demonstrating how the abstract system can reach the target state.

Overapproximation admits behaviours of the system that are not feasible in the original one and the counterexample may not be a valid trace in the real system, so it has to be examined. If it turns out to be a feasible counterexample, the target state is reachable. Otherwise the abstract system has to be refined – hidden details of the original system have to be reintroduced to the model. The goal of the refinement is to modify the abstract system so that it remains an abstraction of the original one, but the spurious counterexample is eliminated. Model checking is performed on the refined system, and the CEGAR loop starts over.

The algorithm terminates when no more counterexamples are found or when a feasible trace is presented leading to the erroneous state.

Example

There are many ways CEGAR can be implemented. An approach is presented here that can be applied to a wide range of formalisms.

When constructing a CEGAR-based algorithm the most important decision to make is the way abstraction is applied to the system. It determines the class of algorithms that can be used for model checking, the nature of the counterexample, and how it can be analyzed, and the possibilities for refinement.

Abstraction can be applied to many aspects of a model. An abstraction method commonly used for formalisms operating on variables is to reduce the number of variables in the model, by simply ignoring some of them [7]. The initial abstraction of the model can be the same model without any variables.

Example 2.5 Consider again automaton $\mathcal{A}$ depicted in Figure 2.2. Eliminating all variables results in the previous finite automaton depicted in Figure 2.1.

Model checking can be performed by state space exploration for example, however, there are several other ways.

Example 2.6 In the first iteration model the state space exploration of $\mathcal{A}$ will result in $G(\mathcal{A})$, where the counterexample found can be e.g. $\sigma = start \rightarrow loop \rightarrow end$.

A common way of checking whether a counterexample is feasible is by transforming it into a first order logic formula, and handing it to a solver [2]. If it is satisfiable, the error location is reachable, and the system is incorrect. Consider for example the case of the extended finite automaton. The counterexample is a trace $\sigma = l_0 \xrightarrow{e_0} l_1 \xrightarrow{e_1} \cdots \xrightarrow{e_n} l_{n+1} = l_{err}$.

Variables have to be defined: for each $x \in \mathcal{V}$ variables $x_0, x_1, \ldots, x_n$ and x_{err} are defined – one for each state on the counterexample. Afer that a set of constraints is constructed to define the automaton's behaviour.

First, initial conditions are defined. In case of extended finite automata, the initial constraints are the variables' initial values. For each $x \in \mathcal{V}$ $x_0 = v_0(x)$ is added to the set of constraints. After that, constraints are added step by step.

It has to be checked if the guard is satisfied. For each $e_i = (l_i, g_i, a_i, l_{i+1})$ $0 \leq i \leq n$ a variation of g_i is added to the set of constraints: all $x \in \mathcal{V}$ appearing in g_i is replaced by x_i. After that, the assignment function is considered: for each $x \in \mathcal{V}$ $x_{i+1} = a_i(x)$ is added to the set of constraint if $a_i(x)$ exists, and $x_{i+1} = x_i$ is added otherwise. Invariants can be turned into constraints the same way as guards: all $x \in \mathcal{V}$ appearing in $I(l_i)$ is replaced by x_i.

Example 2.7 The translation of the counterexample $\sigma = \mathit{start} \rightarrow \mathit{loop} \rightarrow \mathit{end}$ to a logic formula will contain variables x_0, x_1, x_2, l_0, l_1 and l_2. The initial constraints are $x_0 = 0, l_0 = 0$. Edge *start* → *loop* assigns 2 to x, but leaves l unchanged, yielding the constraints $x_1 = 2, l_1 = l_0$. Location *loop*'s invariant $l \leq 5$ can be transformed to constraint $l_1 \leq 5$. The guard of *loop* → *end* yields the constraint $x_1 \geq 16$.

The formula handed to the solver is $\varphi = x_0 = 0 \wedge l_0 = 0 \wedge x_1 = 2 \wedge l_1 = l_0 \wedge l_1 \leq 5 \wedge x_1 \geq 16$.

The conjunction of the resulting set of constraints is handed to a solver. If it is satisfiable, the counterexample is feasible, and the erroneous state is reachable. Otherwise it is a spurious counterexample and the abstract system has to be refined. Solvers can be used for refinement as well, because they often provide additional information that helps choosing the hidden parts of the system that has to be reintroduced to the abstract representation in order to eliminate the counterexample.

Example 2.8 The formula $\varphi = x_0 = 0 \wedge l_0 = 0 \wedge x_1 = 2 \wedge l_1 = l_0 \wedge l_1 \leq 5 \wedge x_1 \geq 16$ is obviously unsatisfiable, because $x_1 = 2 \wedge x_1 \geq 16$ is unsatisfiable in itself. If the automaton's current abstraction is extended with the variable x with all of its assignments, and all constraints in guards and invariants that x appears but l doesn't, this counterexample won't be found anymore.

2.3 Verification of timed systems

The timed automaton is a common formalism for modelling timed systems. It is an extension of the finite automaton with clock variables. In this section clock variables and timed automata are introduced, an algorithm is described (and the implementation briefly explained) for deciding reachability, and information is provided on the complexity of the problem.

2.3.1 Basic definitions

In order to properly define timed automata, first the idea of *clock variables* must be explained. In case of untimed systems, the values of the variables always remain the same between two modifications. However, this is not the case for clock variables (clocks, for short).

Definition 2.5 *Clock variables* are a special type of variables, whose value is constantly and steadily increasing.

When a system stays in one state, the value of clocks are increasing. Naturally, their values can be modified, but the only allowed operation on clock variables is to *reset* them. Resetting a clock means assigning its value to 0. It's an instantaneous operation, after which the value of the clock will continue to increase.

Hereinafter follows some basic definitions that are closely related to clock variables and timed automata.

Definition 2.6 A *valuation* $v(\mathcal{C})$ assigns a non-negative real value to each clock variable $c \in \mathcal{C}$, where $\mathcal{C}$ denotes the set of clock variables.

In other words a valuation defines the values of the clocks at a given moment of time, just like in case of discrete variables.

Definition 2.7 A *clock constraint* is a conjunctive formula of atomic constraints of the form $x \sim n$ or $x - y \sim n$ (*difference constraint*), where $x, y \in \mathcal{C}$ are clock variables, $\sim \in \{\leq, <, =, >, \geq\}$ and $n \in \mathbb{N}$. $\mathcal{B}(\mathcal{C})$ represents the set of clock constraints.

In other words a clock constraint defines upper and lower bounds on the values of clocks (or differences of clocks, in case of difference constraints). Bounds are always integer numbers.

A *timed automaton* extends a finite automaton with clock variables. It can be defined as follows.

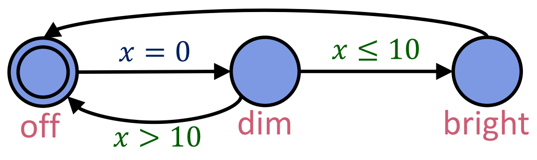


Figure 2.5 Timed automaton model of a lamp

Definition 2.8 A *timed automaton* $\mathcal{A}$ is a tuple $\langle L, l_0, E, I \rangle$ where

- L is the set of locations,
- $l_0 \in L$ is the initial location,
- $E \subseteq L \times \mathcal{B}(\mathcal{C}) \times 2^{\mathcal{C}} \times L$ is the set of edges and
- $I : L \to \mathcal{B}(\mathcal{C})$ assigns invariants to locations. [1]

The automaton's edges are defined by the source location, the guard (represented by a clock constraint), the set of clocks to reset (the timed equivalent of the assignment function), and the target location.

Example 2.9 The timed automaton depicted in Figure 2.5 models a lamp with three levels of intensity: off (no light), dim, and bright. When it is turned off, one push of the button turns the dim light on, two quick (within 10 time units) pushes of the button turn the bright light on. When it is turned on, it can be switched off with a push of the button.

Definition 2.9 A *state* of $\mathcal{A}$ is a pair $\langle l, v \rangle$ where $l \in L$ is a location and $v : \mathcal{C} \to \mathbb{R}$ is the current valuation satisfying $I(l)$.

In the initial state $\langle l_0, v_0 \rangle$ v_0 assigns 0 to each clock variable.
Two kinds of operations are defined.

Definition 2.10 The state $\langle l, v \rangle$ has a *discrete transition* to $\langle l', v' \rangle$ if there is an edge $e(l, g, r, l') \in E$ in the automaton such that

- v satisfies g,
- v' assigns 0 to any $c \in r$ and assigns $v(c)$ to any $c \notin r$, and
- v' satisfies $I(l')$.

Definition 2.11 The state $\langle l, v \rangle$ has a *time transition* (or delay, for short) to $\langle l, v' \rangle$ if

- v' assigns $v(c) + d$ for some non-negative d to each $c \in \mathcal{C}$ and
- v' satisfies $I(l)$.

Example 2.10 The automaton of the previously mentioned lamp operates on one clock variable, x. Initially $x = 0$ but as long as the system stays in location *off*, its value increases and it can reach any non-negative value. Once it steps to *dim*, x is reset, and its value increases from 0 again. The next discrete transition is decided by the amount of time the system spent in location *dim*. If it is at most ten time units, the system steps into location *bright* and continues to increase from its last value in *dim*, that can be any value between 0 and 10. Otherwise, the system steps back to *off* and the value of x continues to increase from its last value in *dim*, that is more than 10.

There are many extensions of the timed automata formalism. Most of them – such as network automata, synchronization, and urgent locations – can be easily transformed into conventional timed automata, but this is not always the case. The idea to allow discrete variables as well as clock variables arises simply, but the same way as in case of finite automaton, growing expressive power yields less efficient model checking.

2.3.2 Timed automaton reachability

In case of timed automata the reachability problem can be defined as follows.

Input : An automaton $\mathcal{A} = \langle L, l_0, E, I \rangle$, and a location $l_{err} \in L$.

Output : An execution trace $\sigma = l_0 \xrightarrow{t_0} l_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} l_{err}$ from l_0 to l_{err} or *No*, if it is unreachable.

This problem is decidable. One of the most effective algorithms for deciding reachability is the algorithm used by *Uppaal*[1], a model checker for timed automata. The core of the algorithm is published in [1].

Algorithm

Before presenting the algorithm, some basic definitions have to be provided. First, zones are introduced as an abstract domain for clock valuations.

[1]http://www.uppaal.org/

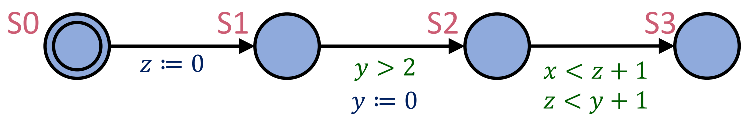


Figure 2.6 Timed automaton

Definition 2.12 A *zone* z is a set of non-negative clock valuations satisfying a clock constraint.

Definition 2.13 A *zone graph* is a finite graph consisting of $\langle l, z\rangle$ pairs as nodes, where $l \in L$ refers to some location of a timed automaton and z is a zone. Edges of the zone graph represent transitions.

A node $\langle l, z\rangle$ of a zone graph represents all states $\langle l, v\rangle$ where $v \in z$. Since edges of the zone graph denote transitions, a zone graph can be considered as an (exact) abstraction of the state space. The main idea of the algorithm is to explore the zone graph of the timed automaton, and if a node $\langle l_{err}, z\rangle$ exists in the graph for some $z \neq \emptyset$, l_{err} is reachable, and the execution trace can be provided by some path-finding algorithm.

The construction of the graph starts with the initial node $\langle l_0, z_0\rangle$, where l_0 is the initial location and z_0 contains the valuations reachable in the initial location by time transitions. Next, for each outgoing edge e of the initial location (in the automaton) a new node $\langle l, z\rangle$ is created (in the zone graph) with an edge $\langle l_0, z_0\rangle \rightarrow \langle l, z\rangle$, where $\langle l, z\rangle$ contains the states to which the states in $\langle l_0, z_0\rangle$ have a discrete transition through e. Afterwards z is replaced by $z^{\uparrow}$ where $z^{\uparrow}$ denotes the set of all valuations reachable from a zone z by time transitions. The procedure is repeated on every newly introduced node of the zone graph. If the states defined by a newly introduced node $\langle l, z\rangle$ are all contained in an already existing node $\langle l, z'\rangle$ ($z \subseteq z'$), $\langle l, z\rangle$ can be removed, and the incoming edge should be redirected to $\langle l, z'\rangle$.

Example 2.11 For ease of understanding the algorithm is demonstrated on the automaton in Figure 2.6. The initial state is $\langle S0, z_0\rangle$ where z_0 is a zone containing only the initial valuation $v_0 \equiv 0$. The initial node is $\langle S0, z_0^{\uparrow}\rangle$, where $z_0^{\uparrow}$ contains all states reachable form the initial state by delay. Since as time passes, the values of the three clocks will be incremented by the same value, x, y and z has the same value in each valuation contained by $z_0^{\uparrow}$. Since there is no invariant in location $S0$ the clocks can take any positive value. Because of this z_0 can be defined by the constraint $x = y = z$ (that is, $x - y = 0 \land y - z = 0$), and the initial node can be defined as $\langle S0; x = y = z\rangle$.

There is only one outgoing transition from the initial location and that resets z,

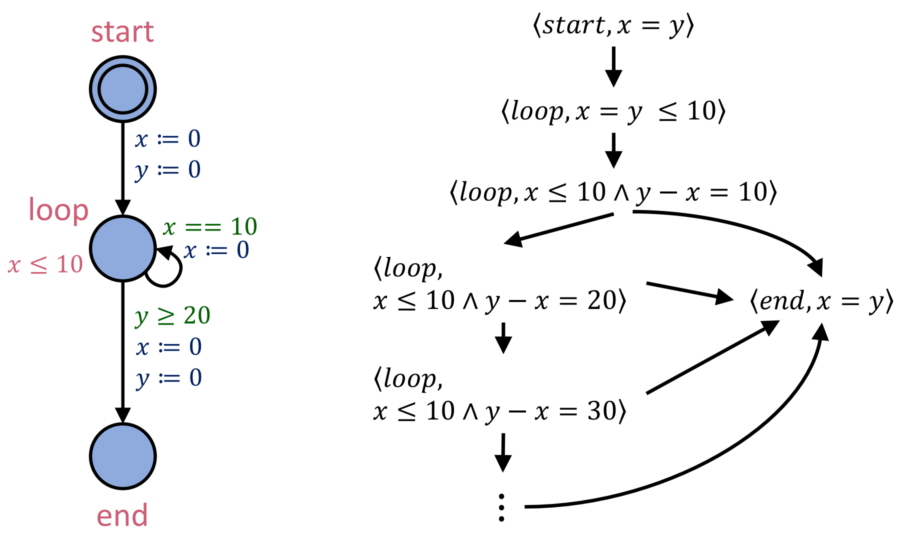

start
$x := 0$
$y := 0$
loop
$x \leq 10$
$x == 10$
$x := 0$
$y \geq 20$
$x := 0$
$y := 0$
end
$\langle start, x = y \rangle$
$\langle loop, x = y \leq 10 \rangle$
$\langle loop, x \leq 10 \land y - x = 10 \rangle$
$\langle loop, x \leq 10 \land y - x = 20 \rangle$
$\langle end, x = y \rangle$
$\langle loop, x \leq 10 \land y - x = 30 \rangle$

Figure 2.7 Timed automaton with infinite zone graph

resulting in the zone defined by $x = y \land z = 0$, which transforms into $z \leq x = y$ when delay is applied. This means the next node of the graph can be defined as $\langle S1, z \leq x = y \rangle$. There is only one outgoing transition from the location $S1$ and it has guard $y > 2$. This means the transition is only enabled in the subzone $z \leq x = y > 2$ (that is $z \leq x \land x = y \land y > 2$). The transition resets y resulting in the zone $y = 0 \land z \leq x > 2$. Delay can be applied and the next node of the graph turns out to be $\langle S2, z \leq x \land y \leq z \land x - y > 2 \rangle$.

The outgoing transition from location $S2$ has a guard $x < z + 1 \land z < y + 1$ from which $x < y + 2$ can be derived contradicting the atomic constraint $x - y > 2$ in the reachable zone of location $S2$. Thus the transition is never enabled, and location $S3$ is unreachable.

The zone graph of this automaton can be drawn as follows.

$$\langle S0; x = y = z \rangle \rightarrow \langle S1, z \leq x = y \rangle \rightarrow \langle S2, z \leq x \land y \leq z \land x - y > 2 \rangle$$

Unfortunately, it is possible that the graph described by the previous algorithm becomes infinite.

Example 2.12 Consider for example the automaton from [1] in Figure 2.7. Constructing the zone graph of this automaton starts similarly, with the node $\langle start, x = y \rangle$. After that both x and y are reset resulting in the zone defined by $x = y = 0$. Location *loop* has an invariant $x \leq 10$ that limits the applicable delay to 10, resulting in $\langle loop, x = y \leq 10 \rangle$, where only the loop-transition is enabled.

The transition resets x resulting in $\langle loop, x = 0 \wedge y = 10 \rangle$. Still only 10 units of delay is enabled, resulting in the node $\langle loop, x \leq 10 \wedge y - x = 10 \rangle$.

From this node, both transitions are enabled. The loop transition increases the difference between x and y yielding the new node $\langle loop, x \leq 10 \wedge y \leq 30 \wedge y - x = 20 \rangle$, while the other transition resets both clocks, resulting in the new node $\langle end, x = y \rangle$.

As we take the new node containing the location $loop$, and apply the loop transition over and over, a new node is always constructed with the difference growing. On the other hand, the other transition always results in $\langle end, x = y \rangle$. Hence the (infinite) zone graph in Figure 2.7.

In order for the zone graph to be finite, a concept called *normalization* is introduced in [1].

Let $k(c)$ denote the greatest value to which clock c is compared in the automaton. For any valuation v such that $v(c) > k(c)$ for some c, each constraint in the form $c > n$ is satisfied, and each constraint in the form $c = n$ or $c < n$ is unsatisfied, thus the interval $(k(c), \infty)$ can be used as one abstract value for c.

Normalization is performed on $z^{\uparrow}$ (before inclusion is checked) in two steps. The first step is removing all constraints of the form $x < m, x \leq m, x - y < m, x - y \leq m$ where $m > k(x)$ (so that x doesn't have an upper bound), and the second step is replacing constraints of the form $x > m, x \geq m, x - y > m, x - y \geq m$ where $m > k(x)$ by $x > k(x), x \geq k(x), x - y > k(x), x - y \geq k(x)$ respectively (to define the new lower bounds).

Example 2.13 In the automaton depicted in Figure 2.7, $k(y) = 20$ (and $k(x) = 10$). This means the exact value of y doesn't really matter, as long as it is greater than 20 – the automaton will behave the exact same way if it is between 30 and 40, or if it is between 40 and 50. If we take this into consideration when constructing the zone graph, the zone $x \leq 10 \wedge y - x = 30$ can be normalized. In this zone, $y \geq 30 > k(y) = 20$, but $x \leq k(x)$. This means we only have to consider constraints bounding y. Implicitly $y \leq 40$ and $y - x \leq 30$. These constraints have to be removed from the zone. Similarly, $y \geq 30$ and $y - x \geq 30$ have to be replaced by $y \geq 20$ and $y - x \geq 20$. The resulting zone is $x \leq 10 \wedge y \geq 20 \wedge y - x \geq 20$. If we replace the original zone $x \leq 10 \wedge y - x = 30$ by this zone, and continue constructing the zone graph, the resulting graph is depicted in Figure 2.8.

Using normalization the zone graph is finite, but unreachable states may appear in it. If the automaton doesn't have any guard or invariant of the form $c_1 - c_2 < n$, the reachability of the location in question will be answered correctly. Otherwise, the algorithm may terminate with a false positive result.

$$\begin{tikzcd}
& \langle \mathit{start}, x = y\rangle \arrow[d] & \\
& \langle \mathit{loop}, x = y \le 10\rangle \arrow[d] & \\
& \langle \mathit{loop}, x \le 10 \wedge y - x = 10\rangle \arrow[dl] \arrow[dr] & \\
\langle \mathit{loop}, x \le 10 \wedge y - x = 20\rangle \arrow[rr] \arrow[dr] & & \langle \mathit{end}, x = y\rangle \\
& \langle \mathit{loop}, x \le 10 \wedge y > 20 \wedge y - x > 20\rangle \arrow[ur] \arrow[loop right] &
\end{tikzcd}$$

Figure 2.8 Finite zone graph

Example 2.14 To demonstrate the incorrectness of the algorithm, consider again the automaton in Figure 2.6. Recall that the reachable states of the automaton (by our calculations) were $\langle S0, x = y = z\rangle$, $\langle S1, z \le x = y\rangle$ and $\langle S2, z \le y \le z \le y \wedge x - y > 2\rangle$ – $S3$ is unreachable. Applying normalization leaves the states $\langle S0, x = y = z\rangle$ and $\langle S1, z \le x = y\rangle$ unchanged, but the normalizing the reachable state in $S2$ results in $\langle S2, z \le y \le z \le y \wedge x - y > 1\rangle$, where the guard can be satisfied, thus making $S3$ reachable.

The operation *split* [1] is introduced to assure correctness. Instead of normalizing the complete zone, it is first split along the difference constraints, then each subzone is normalized, and finally the initially satisfied constraints are reapplied to each normalized subzone. The result is a set of zones (not just one zone like before), which means multiple new nodes have to be introduced to the zone graph (all with edges representing the same transition from the original node).

Example 2.15 To demonstrate the effects of split, let us construct the zone graph of the automaton of Figure 2.6. The original node remains $\langle S0, x = y = z\rangle$, but the next node is first split along the difference constraint $x - z < 1$. Instead of the node $\langle S1, z \le x = y\rangle$, this time there are two nodes: $\langle S1, x = y \wedge x - z < 1\rangle$ and $\langle S1, x = y \wedge x - z \ge 1\rangle$.

From $\langle S1, x = y \wedge x - z < 1\rangle$, $\langle S2, x - z \le 1 \wedge z - y \le 1\rangle$ is reachable, where the transition to location $S3$ is not enabled because of the guard $x - z < 1$.

From $\langle S1, x = y \wedge x - z \ge 1\rangle$ the resulting zone after firing the transition is split along the constraint $z - y < 1$, resulting in nodes $\langle S2, x - z \ge 1 \wedge z - y < 1\rangle$, and $\langle S2, x - z \ge 1 \wedge z - y \ge 1\rangle$. The transition to $S3$ is not enabled in either nodes.

Applying split results in a zone graph, that is a correct and finite representation of the state space [1].

Implementation

Paper [1] also provides an implementation of the zone domain, called *Difference Bound Matrix*, or DBM for short. The idea of DBMs is based on transforming clock constraints to difference logic formulae.

Difference constraints are easy to transform as $c_1 - c_2 \geq n$ is equivalent to $c_2 - c_1 \leq -n$ (same goes for strict inequalities), and $c_1 - c_2 = n$ is equivalent to $c_1 - c_2 \geq n \wedge c_1 - c_2 \leq n$. In order to transform constraints of the form $x < n$ or $x \leq n$, $x \in \mathcal{C}, n \in \mathbb{Z}$ a new variable has to be introduced.

Definition 2.14 The variable denoted by $\mathbf{0}$ is a special variable that has a constant value of 0. $\mathbf{0}$ is not a clock variable, but can appear in clock constraints.

Using $\mathbf{0}$ $x \sim n$, $\sim \in \{\leq, <, =, >, \geq\}$ can be transformed into $x - \mathbf{0} \sim n$, and all clock constraints can be transformed into the desired form.

Definition 2.15 A *Difference Bound Matrix* D of a zone z operating on $\mathcal{C}$ is a square matrix of $|\mathcal{C}| + 1$ rows (and columns). A row and a column is assigned to each $c \in (\mathcal{C} \cup \{\mathbf{0}\})$. Each element $D_{i,j}$ of the matrix describes an upper bound on $i - j$, by storing whether the inequality is strict ($<$ or $\leq$) and the bound n. It is possible that there is no upper bound on $i - j$, in this case $D_{i,j} = \infty$.

The DBM D of zone z stores all constraints bounding z.

In order for operations to be efficient it is required that the DBM is in a *canonical* form.

Definition 2.16 A Difference Bound Matrix D of a zone z is in canonical form if for all $i, j \in (\mathcal{C} \cup \{\mathbf{0}\})$, $D_{i,j}$ denotes the strictest bound on $i - j$ that can be derived from z.

As zones and DBMs are different representations of the same entity, this paper uses the terms interchangeably.

Many operations are defined on DBMs. The most important ones are the following:

- *consistent(D)* is used to decide if D contains any states
- *relation(D,D′)* tells if one of D and D' is contained in the other
- *satisfied(D, m)* where m is a difference constraint, tells if D contains any states satisfying m without affecting D
- *up(D)* calculates $D^{\uparrow}$

- *and(D, m)* where m is a difference constraint, restricts D to the states satisfying m
- *free(D, c)* where $c \in \mathcal{C}$, removes all constraints on c
- *reset(D, c)* where $c \in \mathcal{C}$, resets c
- *norm(D, k)* where $k : \mathcal{C} \to \mathbb{Z}$, normalizes the zone based on k that assigns to each $c \in \mathcal{C}$ the highest value they are compared to in an automaton
- *split(D, $\mathcal{G}$)* where $\mathcal{G} \in \mathcal{B}(\mathcal{C})$, splits the zone based on $\mathcal{G}$ that is the set of all difference constraints appearing in an automaton.

Using these operations the calculation of the next zone in the zone graph can be automated. Let $n = (l, D)$ be an already calculated node of the zone graph, and $e = (l, g, r, l')$ an edge of the timed automaton. Calculating the next node $n' = (l', D)$ (or next nodes) starts by checking guards. This can be performed by calling *and(D,m)* for each atom m of the difference logic formula representation of g and then checking *consistent(D)* to check if there are any states in D satisfying g. If there are, the transition is enabled. In this case D has to be reset: for $c \in r$, *reset(D,c)*. $I(l')$ also has to be satisfied in order for the transition to be enabled. This can be checked similarly to the guard. After this *up* can be used to calculate D', but *and(D,m)* has to be called again for $m \in I(l')$. After that *split* and *norm* are called to ensure correctness and termination.

Implementations (pseudocodes) of these operations are provided in [1]. Termination of the algorithm is also proven, but it's complexity is exponential in the number of clocks. Because of this it is essential to reduce the number of clocks as much as it is possible, without changing the reachability property.

Activity

In [10] abstractions of the automaton are proposed to reduce the number of clock variables without affecting the operation of the automaton. The abstraction that will be used later in this paper is called *activity*. A clock c is considered active at some location l (denoted by $c \in Act(l)$) if its value at the location may influence the future evolution of the system. It might be because the clock appears in the invariant of the location, or in the guard of some outgoing edges of the location, or because it is active in one of the posterior locations and its value is not reset until that location.

Example 2.16 In the automaton depicted in Figure 2.6 clock z is active at location $S2$ because it appears in the guard of the outgoing edge. It is also active in $S1$ because its value in $S1$ determines its value in $S2$ and it is active in $S2$, but it is not active in $S0$ because its value is not important, since it is reset in the outgoing edge anyway.

The core of the algorithm for reducing the number of clock variables is to calculate $Act(l)$ for each $l \in L$, and if $Act(l) < |\mathcal{C}|$ holds for each $l \in L$, the automaton can be reconstructed by *renaming* variables location by location (after renaming there will be less clocks). This is true, even if all $c \in \mathcal{C}$ is active in at least one location, however, clocks might be renamed differently in distinct locations.

Before presenting how activity is calculated some new notations are introduced. Let $clk : \mathcal{B}(\mathcal{C}) \to 2^{\mathcal{C}}$ and assign to each clock constraints the set of clocks appear in it. Define $clk : L \to 2^{\mathcal{C}}$ such that $c \in clk(l)$ iff $c \in clk(I(l))$ or there exist an edge (l, g, r, l') such that $c \in clk(g)$.

Activity is calculated by an iterative algorithm starting from $Act_0(l) = clk(l)$ for each $l \in L$. In the i^{th} iteration $Act_i(l)$ is derived by extending $Act_{i-1}(l)$ by $Act_{i-1}(l') \setminus r$ for each edge (l, g, r, l'). The algorithm terminates when it reaches a fix point, i.e. when $Act_i(l) = Act_{i-1}(l)$ for each $l \in L$.

Example 2.17 Let us calculate activity for the complete automaton. $Act_0(l) = clk(l)$ for each $l \in L$. Thus, iteration starts from $Act_0(S0) = \emptyset$, $Act_0(S1) = \{y\}$, $Act_0(S2) = \{x, y, z\}$, and $Act_0(S3) = \emptyset$.

Since y is not reset on edge $S0 \to S1$, $Act_1(S0) = \{y\}$. Since x and z are not reset on edge $S1 \to S2$, $Act_1(S1) = \{y, x, z\}$. Since $Act_0(S3) = \emptyset$, and $S3$ has no outgoing edges $Act_1(S2) = \{x, y, z\}$ and $Act_1(S3) = \emptyset$.

Clock x is not reset on edge $S0 \to S1$ but z is, thus $Act_2(S0) = \{y, x\}$. Other activities are unchanged in this iterations, thus $Act_2(S1) = \{y, x, z\}$, $Act_2(S2) = \{x, y, z\}$, $Act_2(S3) = \emptyset$ and the fix point is reached.

Complexity

As it was mentioned, reachability for timed automata without discrete variables is decidable (but it it exponential). It was also mentioned before that reachability for finite automata extended by (discrete) variables is undecidable. Obviously, reachability of timed automata extended with discrete variables is also undecidable. However it is decidable if the value sets of the discrete variables are finite, because in this case the values can be encoded in the locations.

2.4 Objectives

The goal of this paper is to provide an extensible framework for CEGAR-based algorithms deciding reachability of timed automata extended with discrete variables. Since reachability is undecidable for this type of timed automata, termination of the algorithms is not always guaranteed.

Chapter 3

Configurable Timed CEGAR

This chapter presents a configurable framework for CEGAR-based reachability analysis of timed automata. First, the goal of the framework is explained, than the used algorithms and techniques are enumerated and finally, the extensibility of the framework presented.

3.1 Generic CEGAR framework

The key idea of the framework is to provide various techniques for each phases of the CEGAR-loop, by using correspondent parts of CEGAR-based reachability algorithms. Most of these algorithms already exist (mostly for other formalisms, and they have to be adapted to timed automata), but some of them are new contributions. The created modules can then be combined to form new algorithms (that is, the technique used in each phase of CEGAR can be provided by different algorithms), and choosing the most efficient parts of the original algorithms can result in an even more efficient algorithm than the original ones.

The architecture of the framework is illustrated in Figures 3.1 and 3.2. There are two different realizations of the CEGAR-loop, because the framework supports two distinct ways to apply abstraction to timed automata. While the first approach (Figure 3.1) is based on eliminating clock variables (see Section 2.2.3): starting from a finite automaton (without any clock variables) the current automaton gets extended with some clocks in each iteration – and the *automaton* is the base of refinement, the other (Figure 3.2) is based on the refinement of the *state space* itself. Only techniques of the same approach are interchangeable – hence the two realizations.

3.1.1 Automaton-based refinement

Figure 3.1 depicts the architecture of the automaton-based approach. The initial abstraction is a finite automaton that is derived from the original timed automaton by

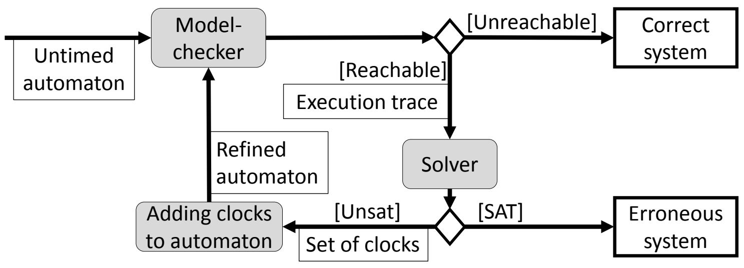


Figure 3.1 Automaton-based refinement

removing all clock variables and clock constraints.

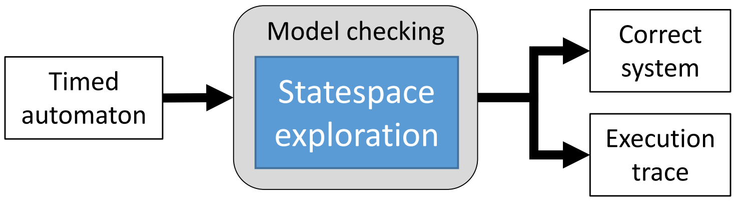
In each iteration of the CEGAR-loop, the task of the model checking phase is to determine whether the error location is reachable in the current automaton and provide a trace (counterexample) if there is one. Therefore, this phase should be realized by a reachability-checking algorithm for timed automata that can find a trace to the location.

The task of the analysis phase is to check if the trace found is feasible in the original automaton and if it isn't, provide a set of clock variables that can then be added to the automaton (with the clock constraints they appear in) so that the model checker won't find this counterexample again. This can be calculated by a solver.

Finally, the only task of the refinement phase is to refine the current abstraction of the automaton, by extending it with the given set of clock variables (and the constraints they appear in). The task is straightforward, and so there is only one technique for this phase of the CEGAR-loop (in case of automaton refinement).

Algorithm 3.1 provides pseudocode for the described approach. Functions *reachable()*, *refinementset()* and *refine()* are components that can be chosen from the techniques described in the following sections, where two reachability algorithms, an approach for calculating the clocks to include, and the algorithm for refining the automaton are presented. Algorithms are demonstrated on the automaton in Figure 2.7 with $l_{err} = end$ to ease understanding. From now on $\mathcal{A}$ will refer to that particular timed automaton.

Zone graph exploration

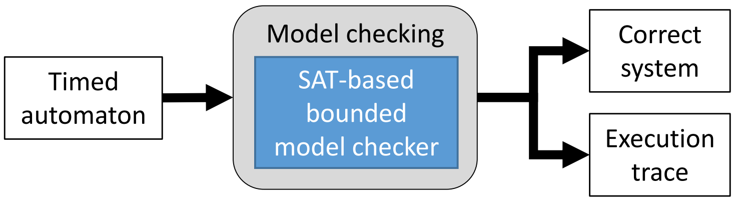


Algorithm 3.1 Automaton-based refinement

```
Input: $\mathcal{A}_\mathcal{C} = \langle L, l_0, E, I \rangle$, $l_{err} \in L$
Output: $\sigma = l_0 \xrightarrow{t_0} l_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} l_{err}$ if $l_{err}$ is reachable, No otherwise
$\mathcal{A} \leftarrow \mathcal{A}_\mathcal{C} \setminus \mathcal{C}$                    /* initial abstraction */
while true do
    $\sigma \leftarrow$ reachable($\mathcal{A}, l_{err}$)                /* model checking */
    if $\nexists \sigma$ then                               /* no counterexample is found */
        return No                                       /* unreachable */
    else
        $C \leftarrow$ refinementset($\mathcal{A}, \sigma$)                /* analysis */
        if $c = \emptyset$ then                    /* No refinement set, because feasible */
            return $\sigma$
        else
            $\mathcal{A} \leftarrow$ refine($\mathcal{A}, C, \mathcal{A}_\mathcal{C}$)      /* automaton is refined by $C$ */
```

The reachability-checking algorithm described in Section 2.3.2 is an obvious choice for the model checking phase, however, it is important to note that the algorithm does not handle discerete variables. The discrete valuation can be encoded into the location (and calculated on the fly) but in this case termination is not ensured (as Section 2.2.1 explains).

Satisfiability-based model checker



Satisfiability-based model checking as introduced in Section 2.2.3 can be directly applied to timed automata – the only necessary change is to define a transformation that can turn a counterexample (an execution trace) into an SMT-problem.

The idea is to separate discrete transitions from time transitions. Consider a counterexample sequence $\sigma = l_0 \xrightarrow{t_0} l_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} l_{err}$. This representation of σ hides the fact that it is important how much time the system spends in each location – i.e. delay transitions. Let us denote the amount of time spent in l_i by d_i. This way σ can be defined by $\sigma = l_0 \xrightarrow{d_0} \xrightarrow{t_0} l_1 \xrightarrow{d_1} \xrightarrow{t_1} \cdots \xrightarrow{d_n} \xrightarrow{t_n} l_{err}$. In this representation $\xrightarrow{d_i}$ can be considered a special kind of transition that increases $v(c)$ for each $c \in \mathcal{C}$ by d_i. Based on this the SMT formula can be constructed.

First, let us assign a variable for each clock in each location, both before and after the delay – that is, this means $2 \cdot n \cdot |\mathcal{C}|$ variables. Let us denote these variables by c_i (for the value of clock c in location l_i before the delay) and c'_i (for the value of clock c in location l_i after the delay). Let us also assign variables for each d_i. The first constraints that have to be added is that each of the defined variables are greater or equal to 0.

The initial constraints can simply be described by $c_0 = 0$ for each $c \in \mathcal{C}$. Delay transitions can be turned into constraints by the following equation $c_i + d_i = c'_i$ for each $c \in \mathcal{C}, 0 \leq i \leq n$. In case of discrete transitions, guards (clock constraints) can be turned into SMT constraints by replacing the clock variables with the defined variables. The guard g_i of a transition $t_i(l_i, g_i, r_i, l_{i+1})$ can be transformed by replacing all clocks c appearing in g_i by c'_i. Resets can also be simply transformed into constraints – for all $c \in r_i$ $c_{i+1} = 0$ has to be added to the set of constraints. Note, that this way c_{i+1} is only specified for the reset clocks. For all $c \notin r_i$ $c_{i+1} = c'_i$ has to be added to the set of constraints. Invariants can be transformed into SMT constraints the same way as guards.

Discrete variables can be mapped to SMT variables as before since discrete variables and clock variables have no affect on each other.

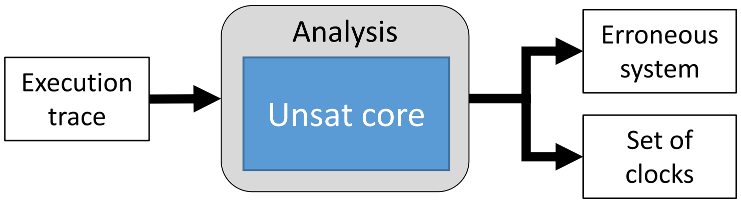
Running example 3.1 Consider automaton $\mathcal{A}$ (recall, it is the automaton depicted in Figure 2.7). Assume the counterexample is $\sigma = start \rightarrow loop \rightarrow end$. The new variables are x_0,y_0,x'_0,y'_0,x_1,y_1,x'_1,y'_1,x_2,y_2,x'_2,y'_2,d_0,d_1 and d_2. It is important to define them to be non-negative since they represent the elapse of time. Initial constraints are $x_0 = 0$ and $y_0 = 0$. Delay is described by constraints $x'_0 = x_0 + d_0$, $y'_0 = y_0 + d_0$, $x'_1 = x_1 + d_1$, $y'_1 = y_1 + d_1$, $x'_2 = x_2 + d_2$ and $y'_2 = y_2 + d_2$.

There is no guard on edge *start* → *loop*, but it resets both variables yielding the constraints $x_1 = 0$, $y_1 = 0$. The invariant of location *loop* can be transformed to $x_1 \leq 10$ and $x'_1 \leq 10$. The next edge has a guard, $y \geq 20$ that can be transformed to $y'_1 \geq 20$, and the resets to $x_2 = 0$ and $y_2 = 0$. This results in the formula $\varphi = x_0 = 0 \wedge y_0 = 0 \wedge x'_0 = x_0 + d_0 \wedge y'_0 = y_0 + d_0 \wedge x'_1 = x_1 + d_1 \wedge y'_1 = y_1 + d_1 \wedge x'_2 = x_2 + d_2 \wedge y'_2 = y_2 + d_2 \wedge x_1 = 0 \wedge y_1 = 0 \wedge x_1 \leq 10 \wedge x'_1 \leq 10 \wedge y'_1 \geq 20 \wedge x_2 = 0 \wedge y_2 = 0$.

This allows us to use a SMT-solver to decide if a possible execution trace of a timed automaton is feasible. This can be used for model checking timed automata, by iterating over all possible execution traces and if a trace σ is found from l_0 to l_{err}, it can be checked, and if the derived formula is satisfiable, σ is proposed as a counterexample.

The problem with this model checker is that there may be infinitely many execution traces. Thus, this model checker can only be used as a *bounded* model checker.

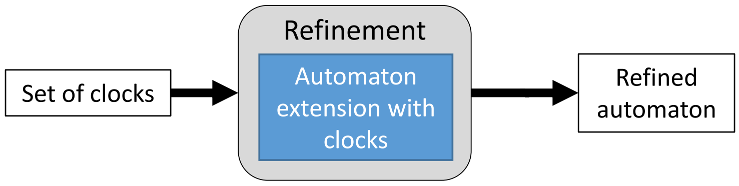
Unsat core-based clock selection



Solvers can be useful, not only to decide if a given set of constraints is satisfiable, but also – if the answer is that the formula is unsatisfiable – solvers have various features to show why they can not be satisfied. One of the possible helpful feature is deriving the so called *unsat core* – that is, a minimal (not necessarily minimum[1]) set of the given constraints that is unsatisfiable in itself. This set of constraints can be used to determine the set of clock variables with what the current abstraction of the automaton has to be extended. In order to define the refinement set, the variables appearing in the unsat core have to be transformed back to the original variables. The set of original variables appearing in the constraints is the result of the algorithm.

Running example 3.2 The result of checking satisfiability of φ is *No*, and the unsat core is $\varphi_{unsat} = x_1' = x_1 + d_1 \wedge y_1' = y_1 + d_1 \wedge x_1 = 0 \wedge y_1 = 0 \wedge x_1 \leq 10 \wedge x_1' \leq 10 \wedge y_1' \geq 20$. This means the automaton has to be refined with both x and y so that the spurious counterexample is eliminated.

Automaton refinement



Given an original automaton $\mathcal{A}$ an abstract automaton $\mathcal{A}'$ and a set of clock variables to be added $C \subseteq \mathcal{C}$, the task is to refine $\mathcal{A}'$ so that each clock $c \in C$ appears in it. The task is to decide which of the guards, resets and invariants to include. Resets are easy to add: the ones that reset clocks in C should be included, others don't. Guards and invariants are clock constraints – conjunctive formulae of atomic constraints bounding

[1]A minimal unsat core does not guarantee that it is the smallest possible unsatisfiable subset of the given constraints, i.e. minimum. It only guarantees that the removal of any constraint would result in a satisfiable formula.

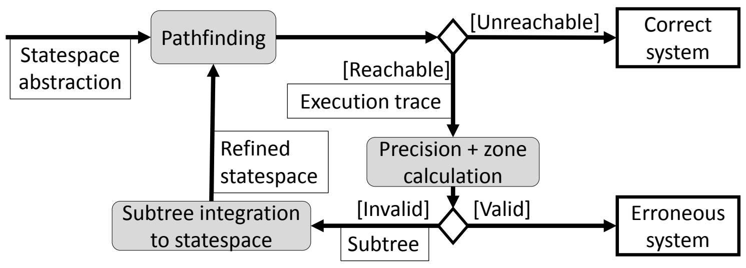


Figure 3.2 State space-based refinement

the value of the clocks or the difference of two clocks. Decision can be made for each atomic formula one by one: those in which only clocks in $\mathcal{A}'$ or C appear – that is, difference constraints are only included if both clocks appear in $\mathcal{A}'$ or C.

3.1.2 State space-based refinement

In case of state space-based refinement, the representation of the state space has a defining role. In the proposed framework, the state space is represented by zone graphs – this is common for all algorithms. However, the abstraction of the zone graph can be performed various ways. In this framework, the main idea is to explore the state space without considering clock variables (and in some cases discrete variables, too), and to refine the state space – trace by trace – by deciding which of the clock variables to include for each of the zones on that path. After that the graph is refined (clocks are included in the zones), and during the refinement it turns out whether the counterexample is feasible or not. Figure 3.2 depicts the architecture of this approach.

Because of the different approaches of abstraction, constructing the initial abstraction is not as straightforward as it was in case of automaton-refinement. All that can be said is that it is some sort of abstraction of the state space derived from the automaton without including clock variables.

The task of the model checking phase is to find a path from the initial location to the error location in the current abstraction of the zone graph. Because of this, the model checking phase of state space-based refinement is performed by pathfinding algorithms.

The task of the analysis phase can be divided into two parts: decide which of the clock variables to include in the zones (i.e. the *precision* of the zone) and calculate the zones on the trace (up to the given precision) and find out if it is feasible or not.

When performing the first part it is important to find precisions (that might change along the trace) that is not too big (does not include too much variables) to be calculated efficiently, but includes all variables that are necessary to find out if he trace is feasible or not. The result of this part should be a function $P : V(G) \rightarrow 2^{\mathbb{C}}$ assigning precisions to the nodes of the current abstraction of zone graph.

Algorithm 3.2 State space-based refinement

```
Input: $\mathcal{A} = \langle L, l_0, E, I \rangle$, $l_{err} \in L$
Output: $\sigma = l_0 \xrightarrow{t_0} l_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} l_{err}$ if $l_{err}$ is reachable, No otherwise
$\mathcal{G} \leftarrow$ inital($\mathcal{A}$)                                  /* initial abstraction */
while true do
    $\sigma \leftarrow$ path($\mathcal{G}, l_0, l_{err}, \mathcal{A}$)                    /* model checking */
    if $\nexists \sigma$ then                                 /* no counterexample is found */
        return No                                                    /* unreachable */
    else
        $P \leftarrow$ prec($\mathcal{A}, \sigma$)
        $T \leftarrow$ calculateTrace($\sigma, P$)                                 /* analysis */
        if $\exists \langle l_{err}, z \rangle \in$ nodes($T$) for some $z \neq \emptyset$ then      /* feasible trace */
            return $\sigma$
        else
            $\mathcal{G} \leftarrow$ refine($\mathcal{G}, T$)               /* state space is refined accoring to $T$ */
```

As for the second part, calculating the correct zones can be performed by the steps of the algorithm presented in Section 2.3.2 with some modifications that help with handling the changes of precision along the zones in counterexample. If the error location is unreachable, a guard or invariant will eventually prove to one of the edges on the trace that it represents a transition that is not enabled.

The task of the refinement phase is to modify the current abstraction of the zone graph according to the states calculated in the analysis phase.

Algorithm 3.2 provides pseudocode for the described approach. Function *initial*, *path* and *refine* are state space representation-dependent. Functions *prec()*, and *calculateTrace()* are components that can be chosen from the techniques described in the following sections, where two abstractions of the zone graph are presented, and algorithms are shown, mentioning the state space representation-dependent behaviours of model checking and refinement. Only those of the presented techniques can be used interchangeably, that are defined for the same representation.

Graph representation

The first representation of the abstract zone graph is another zone graph, with zones of varied precisions. To avoid confusion, from now on precisions of zones will always be shown: zones will be denoted by z_C where $C \subseteq \mathcal{C}$ is the precision of the zone. Zones of the real zone graph (without abstraction) are denoted by $z_{\mathcal{C}}$.

$$\begin{tikzcd}
\langle start, z_\emptyset \rangle \arrow[d] \\
\langle loop, z_\emptyset \rangle \arrow[d] \arrow[loop right] \\
\langle end, z_\emptyset \rangle
\end{tikzcd}$$

Figure 3.3 Initial abstraction of $\mathcal{A}$

A node $\langle l, z_C \rangle$ of the abstract zone graph can represent any nodes $\langle l, z'_{\mathcal{C}} \rangle$ of the real zone graph, that contains the same location l, and some zone $z'_{\mathcal{C}}$ for which $z'_C \subseteq z_C$ holds (where z'_C means a spatial projection of $z'_{\mathcal{C}}$ to the subspace spanned by the clocks in C). This means $\langle l, z_\emptyset \rangle$ can represent any nodes of the real zone graph containing l.

Based on this, the initial abstraction can be constructed by assigning a node $\langle l, z_\emptyset \rangle$ to each location $l \in L$. The graph can then be completed with edges: for each $e = (l, g, r, l') \in E$ a new edge of the zone graph should be included pointing from $\langle l, z_\emptyset \rangle$ to $\langle l', z_\emptyset \rangle$.

Running example 3.3 The initial abstraction for $\mathcal{A}$ is depicted in Figure 3.3.

During the algorithm this graph will be refined by the zones calculated in the refinement phase. Sometimes nodes will get replicated, or edges deleted (the precise algorithm will be described later), but it will remain an abstraction of the concrete zone graph.

Tree representation

The other representation of the abstract zone graph is based on the idea of search trees. Instead of keeping track of the full (abstract) zone graph (like we did with the other representation) details of the tree will be uncovered in the model checking phase of the CEGAR loop. However, one thing is common in both representations: the abstraction of the nodes is based on a set of clocks (precision) to include (just like in case of the automaton-based refinement) and initially all precisions are empty. The state space exploration will also operate on empty precision sets, and the zones will be calculated in the refinement phase. In this case, discrete valuations can be calculated during state space exploration (but it is not necessary).

Let us define the formalism to represent the abstract tree.

Definition 3.1 The auxiliary graph can be defined as a tuple $\langle N_e, N_u, E^{\uparrow}, E^{\downarrow} \rangle$ where

- $N_e \subseteq L \times \mathcal{B}(\mathcal{C})$ is the set of explored nodes,
- $N_u \subseteq L \times \mathcal{B}(\mathcal{C})$ is the set of unexplored nodes,
- $E^{\uparrow} \subseteq (N_e \times N)$, where $N = N_e \cup N_u$ is the set of upward edges and
- $E^{\downarrow} \subseteq (N_e \times N)$ is the set of downward edges.

The sets N_e and N_u as well as the sets $E^{\uparrow}$ and $E^{\downarrow}$ are disjoint. $T^{\downarrow} = (N, E^{\downarrow})$ is a tree.

Nodes are built from a location and a zone like in the zone graph but in this case nodes are distinguished by their trace leading to them from the initial node. This means the graph can contain multiple nodes with the same zone and the same location, if the represented states can be reached through different traces. The root of T is the initial node of the (abstract) zone graph. A downward edge e points from node n to n' if n' can be reached from n in one step in the zone graph.

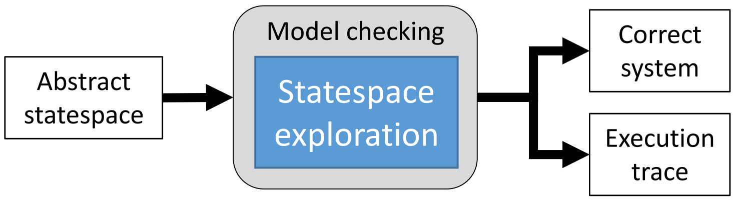
Upward edges are used to collapse infinite traces of the representation, when the states are explored in former iterations. An upward edge from a node n to a previously explored node n' means that the states represented by n are a subset of the states represented by n', thus it is unnecessary to keep searching for a counterexample from n, because if there exists one, another one will exist from n'. Searching for new traces is only continued on nodes without an upward edge. This way, the graph can be kept finite, unless the discrete variables of the automaton prevent it.

Initially, the graph contains only one, unexplored node $\langle l, z_{\emptyset} \rangle$, and as the state space is explored, unexplored nodes become explored nodes, new unexplored nodes and edges appear, until a counterexample is found. During the refinement phase zones are calculated, new nodes and edges appear and complete subtrees disappear. State space exploration will then be continued from the unexplored nodes, and so on. Discrete valuation can be calculated during state space exploration.

Running example 3.4 The initial abstraction for $\mathcal{A}$ is a single unexplored node $n_0 = \langle start, z_{\emptyset} \rangle$.

State space exploration

The task of the model checking phase is to find traces from l_0 to l_{err}. In case of the graph representation, where l_{err} appears in the node $\langle l_{err}, z_{\emptyset} \rangle$ even in the initial abstraction, model checking becomes a pathfinding problem from $\langle l_0, z_{\emptyset} \rangle$ to $\langle l_{err}, z_{\emptyset} \rangle$ in the abstract



zone graph. This can be performed by any pathfinding algorithm.

> **Running example 3.5** Pathfinding in $\mathcal{A}$'s initial abstraction finds the previously mentioned counterexample $\sigma = start \rightarrow loop \rightarrow end$.

In case of the tree representation, l_{err} does not appear in the graph and the state space exploration has to be continued until a node $\langle l_{err}, z_\emptyset \rangle$ appears. State space exploration has to be performed the following way.

In each iteration a node $n = \langle l, z_C \rangle \in N_u$ for some C is chosen. First, it is checked if the states n represents are included in some other node $n' = \langle l, z'_C \rangle$ with a zone of the same precision. If this is the case an upward edge is introduced from n to n' and n becomes explored. Otherwise, n has yet to be explored. For each outgoing edge $e(l, g, r, l')$ of l in the automaton a new unexplored node $\langle l, z_\emptyset \rangle$ is introduced with an edge pointing to it from n, which becomes explored. If any of the new nodes contains l_{err}, the algorithm terminates. Otherwise, another unexplored node is chosen, and so on.

> **Running example 3.6** Exploration of $\mathcal{A}$'s abstract state space starts by exploring n_0, that is performed by introducing a new (unexplored node) $n_1 = \langle loop, z_\emptyset \rangle$ with a downward edge $n_0 \rightarrow n_1$. The erroneous location is not explored yet so the iteration continues. Location *loop* has two outgoing edges. The loop edge introduces $n_2 = \langle loop, z_\emptyset \rangle$ with a downward edge $n_1 \rightarrow n_2$. Edge $loop \rightarrow end$ introduces $n_3 = \langle end, z_\emptyset \rangle$ with $n_1 \rightarrow n_3$. The erroneous location is found and $\sigma = n_0 \rightarrow n_1 \rightarrow n_3$ is proposed as a counterexample.

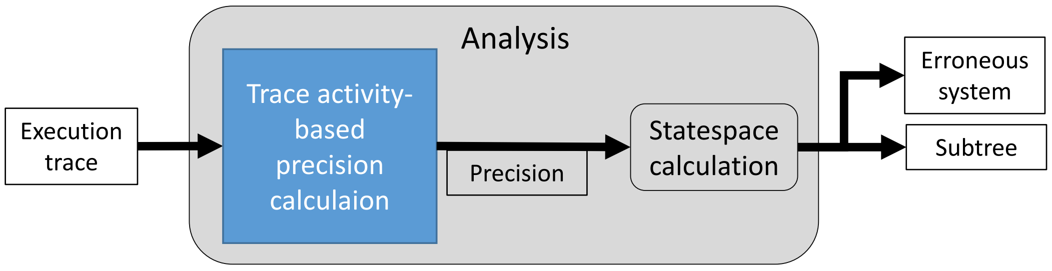
Algorithm 3.3 provides pseudocode for the presented method. Function *trace* calculates the sequence of downwards arrows through wich the node is reached from the root of $\mathcal{T}$.

Trace activity-based precision calculation

The task of the analysis phase is to determine the precision of each zones on a given counterexample. The abstraction *activity* as described in Section 2.3.2 is able to assign a set of clocks for each locations of the automaton, without affecting its behaviour.

Algorithm 3.3 An implementation of of function *path*

```
Input: $\mathcal{T} = \langle N_e, N_u, E^{\uparrow}, E^{\downarrow} \rangle$, $l_{err}$, $\mathcal{A}$
Output: $\sigma = l_0 \xrightarrow{t_0} l_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} l_{err}$ if $l_{err}$ is reachable in the abstract statespace, $\emptyset$
        otherwise
for $n = \langle l, z_C \rangle \in N_u$ do
    newnodes $\leftarrow \emptyset$                                   /* set of successor nodes */
    $N_u \leftarrow N_u \setminus \{n\}$
    $N_e \leftarrow N_e \cup \{n\}$                                  /* node marked as explored */
    if $\exists n' = \langle l, z'_C \rangle \in N_e$ such that $z_C \subseteq z'_C$ then   /* $n'$ contains all states of $n$ */
        $E^{\uparrow} \leftarrow E^{\uparrow} \cup \{(n, n')\}$
    else
        for $e = (l, a, g, l') \in E(\mathcal{A})$ do
            $n' \leftarrow \langle l', z_{\emptyset} \rangle$
            newnodes $\leftarrow$ newnodes $\cup \{n'\}$
            $N_u \leftarrow N_u \cup \{n'\}$
            $E^{\downarrow} \leftarrow E^{\downarrow} \cup \{(n, n')\}$
    if $\exists n' = \langle l_{err}, z_{\emptyset} \rangle \in$ newnodes then              /* $l_{err}$ reached */
        return trace$(n')$
return $\emptyset$                                             /* $l_{err}$ is unreachable */
```



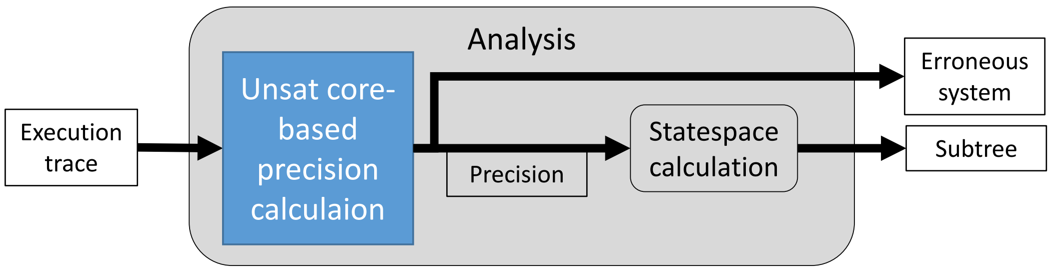
Assigning $act(l)$ for each node $n = \langle l, z_C \rangle$ would be a good solution of the task, however it can be made more efficient by considering the fact that we are only examining an execution trace, and we only need to know if it is feasible.

Based on *activity* a new abstraction can be introduced, called *trace activity* $Act_{\sigma}(n) : N \rightarrow 2^{\mathfrak{C}}$ which does the same thing as activity, except for a trace: it assigns precisions to nodes (not locations in this case, because the same location may appear multiple times ion a trace with different activity). The algorithm calculating trace activity operates the following way.

The algorithm iterates over the counterexample trace, but backwards. In the final node $n_{err} = \langle l_{err}, z_\emptyset \rangle$ it is not important to know the valuations, as the only important thing to know if it is reachable. Therefore $Act_\sigma(n_{err}) = \emptyset$. After that $Act_\sigma(n_i)$ can be calculated from $Act_\sigma(n_{i+1})$ and the edge $e_i(l_i, g_i, r_i, l_{i+1})$ used by transition t_i. Since r_i resets clocks, their values in l_i will have no efficient on the systems behaviour in l_{i+1}. Thus clocks in r_i can be excluded. It is necessary to know if t_i is enabled, so $clk(g_i)$ must be active in n_i. It is also important to satisfy the invariant of l_i thus $clk(I(l_i))$ must be included. This gives us the formula $Act_\sigma(n_i) = (Act_\sigma(n_{i+1}) \setminus r_i) \cup clk(g_i) \cup clk(I(l_i))$.

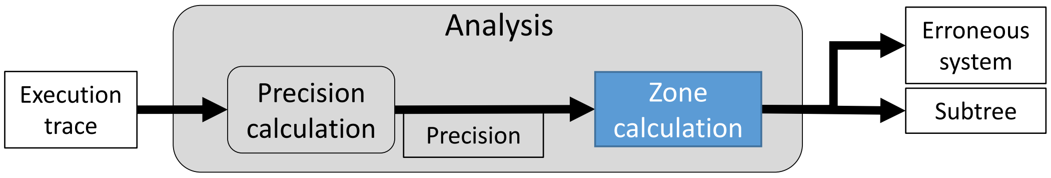
Running example 3.7 Calculating trace activity of $\sigma = n_0 \to n_1 \to n_3$ starts from the erroneous node n_3 by $Act_\sigma(n_3) = \emptyset$. Both x and y are reset on edge *loop* $\to$ *end* and only y is tested, but x appears in *loop*'s invariant, thus $Act_\sigma(n_1) = \{x, y\}$. Since both clocks are reset on edge *start* $\to$ *loop*, $Act_\sigma(n_0) = \emptyset$.

Unsat core-based precision calculation



Unsat core can also be used to determine the necessary precision of a given counterexample. First, the SMT formula described in Section 11 is checked by a solver. If it is satisfiable, the counterexample is feasible. Thus, there is no need to refine the graph, the CEGAR algorithm can terminate (or $\emptyset$ can be assigned to all nodes as a precision and the algorithm will terminate in the refinement phase). Otherwise, unsat core has to be examined. When constructing the SMT formula, variables were introduced for each step. Thus precision can be obtained from the unsat core by step: if c_i or c_i' appears in the unsat core c must be included in the precision assigned to n_i.

Running example 3.8 As it was mentioned, the unsat core of φ is $\varphi_{unsat} = x_1' = x_1 + d_1 \wedge y_1' = y_1 + d_1 \wedge x_1 = 0 \wedge y_1 = 0 \wedge x_1 \leq 10 \wedge x_1' \leq 10 \wedge y_1' \geq 20$. The included variables representing clock variables are x_1', x_1, y_1' and y_1. This results in precision $\emptyset$ for n_0, $\{x, y\}$ for n_1 and $\emptyset$ for n_3.



State space calculation

The task of the refinement phase is to assign correct zones of the given precision for each node in the trace. It is important to mention that the zones on the trace may already be refined to some precision C' that is independent from the new precision C. In this case the zone has to be refined to the precision $C \cup C'$. The initial zone can be calculated as described in Section 2.3.2, except this time not all variables have to be included. After that for each edge in the trace, the zone in the next node can be calculated with some little modifications of the corresponding part of the zone graph exploration algorithm regarding the precision change.

Assume the zone z_i of node n_i is refined to precision C_i and the next zone z_{i+1} in node n_{i+1} has to be refined to C_{i+1}. Consider the DBM implementation of zones. Variables $C_{old} = C_i \setminus C_{i+1}$ have to be excluded from the precision. This can be done by performing *free(c)* for each $c \in C_{old}$, but in [1] the operation *free(c)* only affects the row and the column belonging to c. Thus, for space saving purposes, the row and column of c can simply be deleted from the DBM.

Variables $C_{new} = C_{i+1} \setminus C_i$ have to be introduced. This is a more complex task, since the value is necessary to know. *Trace activity* is constructed in a way that new clocks can only appear when they are reset. In this case, introducing the new variable is simple: add a new row and column to the DBM, belonging to c and call *reset(c)*. However this is not always the case for *unsat core*. It is possible that some constraints only appear in the unsat core, because they contradict each other, or a variable c may appear in the unsat core, because several constraints combined can result in an unsatisfiable constraint that does not include c.

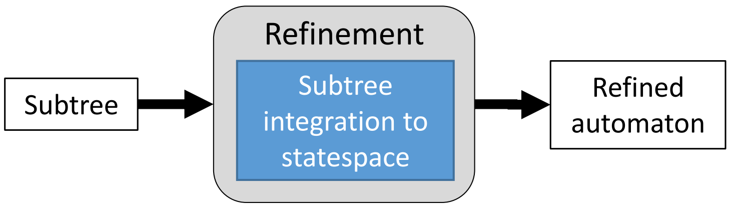
Example 3.9 Consider the automaton in Figure 2.6. The unsat core-based precision of $S1$ is $\{x, y\}$, and $\{x, y, z\}$ of $S2$, but z is not reset on $S1 \rightarrow S2$. In the unsat core it appears in $x < z + 1$ and $z < y + 1$ that imply $x < y + 2$.

It is clear that in this case the concrete value of the variable z doesn't matter, it is only there so that the constraints it appears in are considered. Because of this, there is no need to assign a precise value to z – introduce a row and a column belonging to z and then call *free(z)*.

The correct zones on the trace are calculated. It is important to consider that sometimes the *split()* operation results in more than one zones. In this case the corresponding node is replicated and one of the result zones is assigned to each versions of the node. Exploration has to be continued from that node, thus the refinement of a trace may result in a tree.

Running example 3.10 The refinement of trace σ results in $\langle start, z_\emptyset \rangle \rightarrow \langle loop, x = y \leq 10 \rangle$.

State space refinement



The next important question is how to integrate the refined tree to the graph. The answer depends on which representation is used.

In case of the graph representation integrating has to be done carefully. Before changing the abstract zone to the refined one we must consider the other incoming edges of the node. The states reachable from that edge may not be contained in the refined zone, and thus if there is an an edge pointing to the node to refine other than the one in the trace, the node should be duplicated, and the other incoming edges should be pointing to the new node (that doesn't get refined). Also, if the result of *split()* is multiple zones, the node has to be replicated, but this time no edges has to be redirected, and one of the refined zones can be assigned to each nodes.

Discrete valuation also has to be calculated at this point. The same discrete valuation has to be assigned for each replicas of the node.

The next step is checking containment. Suppose at one point of the algorithm the zone z_C in node n is refined to $z_{C'}$ which is a subzone of a zone $z'_{C'}$ in a node n' containing the same location. In this case any state that is reachable from n is also reachable from n', thus any edge leading to n can be redirected to n', and n can be removed.

If the erroneous location is reachable through this path, the procedure finds it, and the CEGAR algorithm terminates. Otherwise, at some point a guard or a target invariant is not satisfied – the transition is not enabled. The corresponding edge is removed and the analysis of the path terminates.

Running example 3.11 Integration of the refined trace to the abstraction of the zone graph starts from the first node. Since the zone is still $z_\emptyset$, there is not much to be done. The next node on the trace is $\langle loop, z_\emptyset \rangle$. The loop edge is an incoming edge to the node itself. Thus, the node is duplicated and the loop-edge from the original one is redirected to the new one. Now the zone in the original node can be refined to $x = y \leq 10$. Since *loop* $\rightarrow$ *end* is not enabled on the trace, the outgoing edge from $\langle loop, x = y \leq 10 \rangle$ is removed from the graph.

The graph after the process looks as follows.

$$\langle start, z_\emptyset \rangle \rightarrow \langle loop, x = y \leq 10 \rangle \rightarrow \langle loop, z_\emptyset \rangle \rightarrow \langle end, z_\emptyset \rangle$$

Incoming edges that are not on the trace are also important in case of tree representation, however, because of the tree nature of T, the other incoming edges of a node n can only be upwards edges, representing that all states represented by some node n' are also represented by n. Obviously, this may not be true, after refining the zone in the node, and because of this the edge $n' \rightarrow n$ is removed, and n' is marked as unexplored.

Since T is already a tree, it does not cause problems to attach new subtrees to it (because of *split*), but all new nodes have to be marked as unexplored, since only one outgoing edge (of the automaton) were considered when calculating the new subtree, and there could be more.

Containment can also be checked here, just as in case of the graph representation, but it only matters for the leaves of the tree (since the other nodes are already explored). The other possibility is to mark the leaves unexplored and state space exploration will search for containment.

Running example 3.12 Since there are no upwards edges in the current tree, refinement can be performed by replacing the zone in n_1 with the refined one and removing edge $n_1 \rightarrow n_3$ resulting in the following graph.

$$\langle start, z_\emptyset \rangle \rightarrow \langle loop, x = y \leq 10 \rangle \rightarrow \langle loop, z_\emptyset \rangle$$

3.2 Result

Figure 3.4 depicts the presented techniques for automaton-based refinement and Figure 3.5 depicts the presented techniques for state space refinement. In the latter case state space representation-dependent modules are marked with the same colours. Precision and zone calculation are not state space representation-dependent algorithms and can be combined with any of the other colours.

The presented framework is extensible in may ways. New techniques can be added to the framework and combined with existing ones, e.g. an online pathfinding algorithm can be studied in place of the current depth first search algorithm represented by the

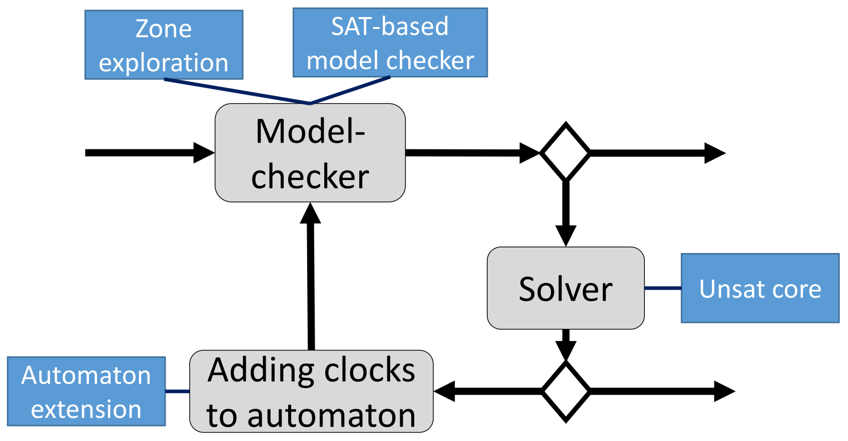


Figure 3.4 Automaton-based refinement

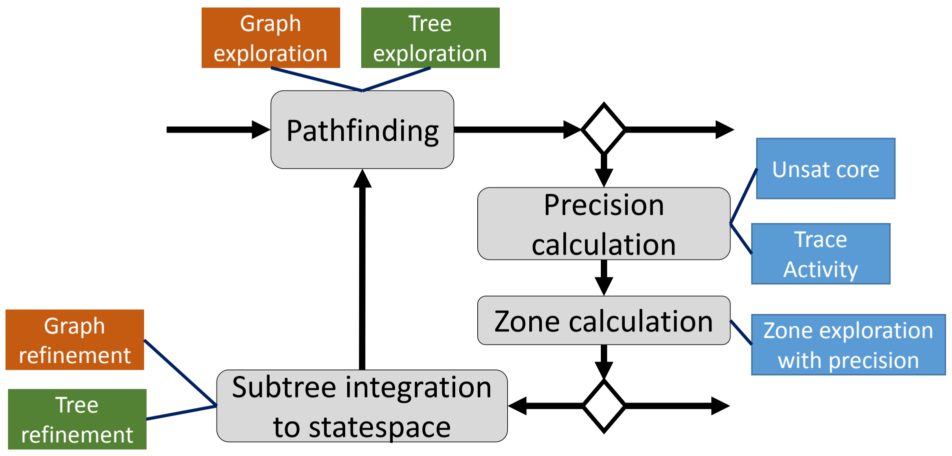


Figure 3.5 Statespace-based refinement

graph exploration module. New representations can be defined for state space-based refinement (e.g. zones can be represented by *potential graphs* [12]), and even a new aspect of abstraction can be introduced with it's own CEGAR-loop realization (e.g. abstraction can be applied to locations [17]).

A total of six algorithms can be composed of the presented techniques. Two of them apply abstraction to the clock variables of the automaton, similarly to the algorithms presented in papers [11, 15, 21], while the other four apply abstraction directly to the state space.

Chapter 4

Implementation

4.1 Environment

4.1.1 The theta framework

theta is an extensible, configurable verification framework developed by the Department of Measurement and Information Systems that offers model checking algorithms for various models, such as programs and statecharts. The models are described by domain specific languages, and translated to common formalisms, including the state transition system, the control flow automaton, and the timed automaton. Besides formalisms, abstract domains and frameworks for common model checking approaches are also implemented. theta uses an SMT solver called Z3[1], that is able to recognize various first order theories, such as difference logic.

I have decided to extend the theta framework with the presented configurable framework for model checking timed automata. The implemented framework relies on the model checker's extended timed automaton representation: the *Timed Control Flow Automaton*, Z3 interface, and a modified version (modifications described in Chapter 3) of the zone implementation described in [1].

4.1.2 Achitecture

The basic architecture of the framework presented in Chapter 3 is shown in Figure 4.1.

The input of the algorithm consists of an input of the problem (a timed automaton $\mathcal{A}$ and a location $l_{err} \in L(\mathcal{A})$), and a configuration of the algorithm: compatible implementations of the CEGAR phases, and their parameters (e.g. the bound of the bounded model checker).

The output of the algorithm can be an execution trace by which l_{err} is reachable, *No* if l_{err} is unreachable, or *Undecided*. The latter case can happen for two causes:

[1]https://github.com/Z3Prover

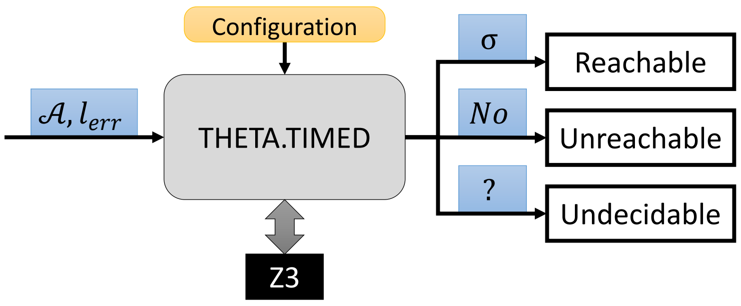


Figure 4.1 Basic architecture of the framework

either the computations on the discrete variables make the problem undecidable, or the bounded model checker proved that l_{err} is unreachable in the given number of steps.

4.2 Measurements

The goal of the measurements is to evaluate the designed algorithm's performance and scalability, and draw conclusions about what combination of algorithms are efficient. The inputs are scalable automata chosen from Uppaal's benchmark data[2]. Uppaal supports extensions of the timed automaton formalism (network automata with synchronization channels) that are not implemented in the theta framework, but can be transformed to the timed control flow automata formalism. This transformation was performed before the measurements.

Measurements were performed on a personal computer with a 2.60 GHz core i5 processor. The program was operating on a maximum of 4GB memory, however, this was not fully used, as it was the solver that run out of memory in most cases.

4.2.1 Inputs

This section describes the input models used for measurements. These models are widely used in benchmarks of timed automata-related algorithms.

Fischer's protocol

Fischer's protocol assures mutual exclusion by bounding the execution times of the instructions. It can be applied to a number of processes accessing a shared variable. Figure 4.2 shows the operation of a process. The location *critical* indicates that the process is in the critical section. The value of the shared variable id ranges between

[2]https://www.it.uu.se/research/group/darts/uppaal/benchmarks/

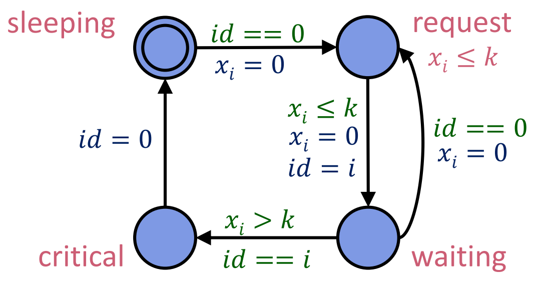


Figure 4.2 Fischer's protocol

0 and n, where n denotes the number of processes. The model also contains a clock variable x_i for each process where $i \in \{1 \dots n\}$ denotes the identifier of the process. The constant k is a parameter of the automaton.

The examined property (mutual exclusion) can be formulated as an input of the reachability problem, where the system is network of n instances of the depicted automata and the reachable states are when at least two of them are in the critical section.

CSMA/CD protocol

Carrier sense multiple access with collision detection (CSMA/CD) is a media access control method used in Ethernet technology. The stations are communicating through a medium that can only maintain one transmission. They can sense if the medium is busy, but there is a certain amount of propagation delay (denoted with σ), so it is possible that another station started transmission since the last information, and collision can occur. In this case the medium broadcasts a jam signal, and the stations pick a random time between 0 and 2σ time units to try transmission again.

The examined property (collision detection) can be formulated as an input of the reachability problem, where the system consists of one medium and n stations ($n \geq 2$) and the reachable states are when at least two of them are transmitting and at least one of them has been transmitting since at least 2σ time units – i.e. the collision was not detected.

Token ring FDDI protocol

Token ring and FDDI protocal are actually two distinct protocols, that are based on the same idea: a ring network of server stations, with a token travelling around the ring. The server owning the token is allowed to communicate, but only within certain time limits to ensure fairness: first, synchronous transmission happens, that is only allowed for at most *sa* time limits (*sa* is the previously agreed timebound of synchronous communication) and then assynchronous transmission can happen until the station exceeds *ttrt*, which is the *tartget token rotation time*: the time passed since the previous transmission of the same station.

The examined property (token exclusiveness) can be formulated as an input of the reachability problem, where the system consists of one token ring and n stations ($n \geq 2$) and the reachable states are when at least two of them are owning the token.

4.2.2 Results

This section presents the performed measurements and their results.

Token ring FDDI measurements

The Token ring protocol is a special input, since the examined safety property can be proven solely based on the structure of the automaton, thus the analysis of the initial abstraction is able to prove the property and refinement is not necessary. This proves how useful abstraction is, but the measurements on this automaton can only compare the efficiency of the pathfinding algorithms, that are almost the same in all presented algorithms (breadth-first search and depth-first search).

The algorithms were executed on the timed automata representations of token rings consisting of 2^n stations ($2 \leq n \leq 11$), and their execution times were measured. Each algorithms on each input was run 15 times and the average the execution times was calculated. The results are depicted in Figure 4.3. The execution times were roughly the same, all in $\mathcal{O}(n^2)$, as it is expected from pathfinding algorithms, however, these measurements did not include the algorithm that uses a bounded model checker.

Bounded model checker measurements

The algorithm that uses a bounded model checker can not be compared to the other algorithms, because it scales with different characteristics. To demonstrate this I have executed the algorithm with on token ring protocol of different sizes, 15 times each and calculated the average runtime. The bound was set to 50 each time.

Figure 4.4 depicts the result of the measurements. It can be seen, that the execution times do not grow (although it is interesting how the execution times decrease – this is probably caused by some environmental characteristics). On the other hand, the bound has a great influence on the runtime. To demonstrate this, I have executed the algorithm with different bound values on the timed automaton model of the token ring of three stations – 15 times each. The bound varied between 5 and 50.

The results of the measurements are depicted in Figure 4.5. The scale on the vertical axis is logarithmic, and the trendline is parabolic. This means that the complexity of this algorithm is greater, than exponential in the value of the bound.

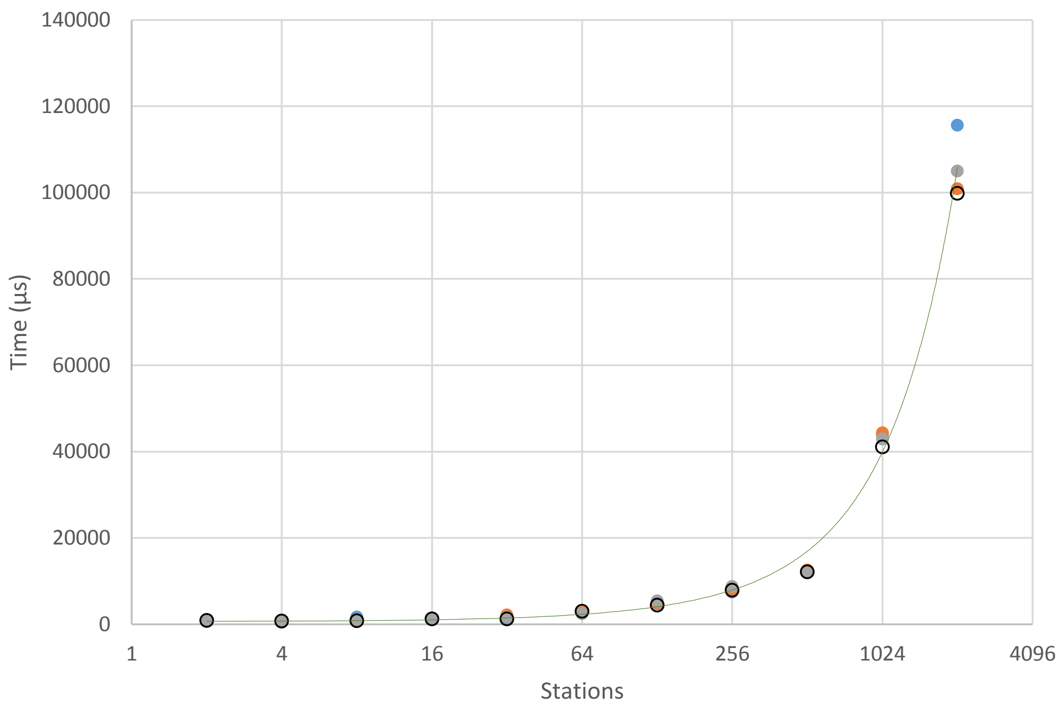


Figure 4.3 Results of measurements on the token ring example

Measurements on Fischer and CSMA/CD protocols

The other algorithms were measured on timed automata representations of Fischer and CSMA/CD protocols, six times each. Memory problems occured at the Fischer protocol of four processes and the CSMA/CD protocol of five stations. One of the algorithms couldn't run on the CSMA/CD protocol of four stations either.

The results of the measurements are depicted in Figure 4.6. The models are denoted by Fisch*n* and CSMA*n* for the Fischer protocol and the CSMA/CD protocol of *n* participants, respectively. The abbreviations denote the algorithms, that are composed the following way:

- AZ denotes abstraction-based refinement with zone graph exploration
- STU denotes state space-based refinement operating on the tree representation of the abstract zone graph where the precisions of the zones are based on the unsat core of the trace,
- SGU denotes state space based refinement operating on the graph representation of the abstract zone graph where the precisions of the zones are based on the unsat core of the trace,

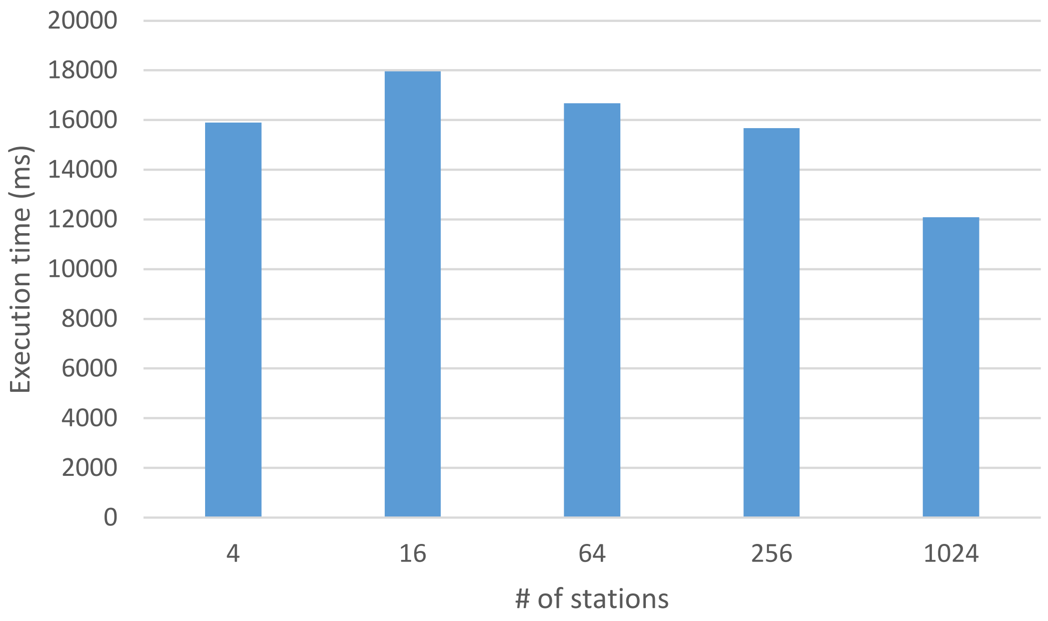


Figure 4.4 Results of measurements with different number of stations

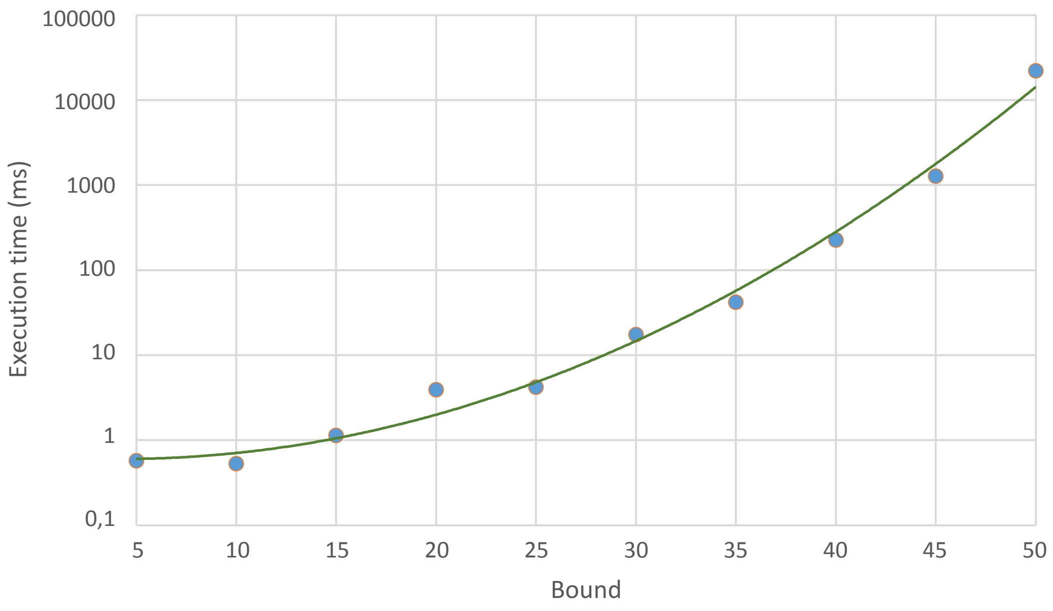


Figure 4.5 Results of measurements with different bounds

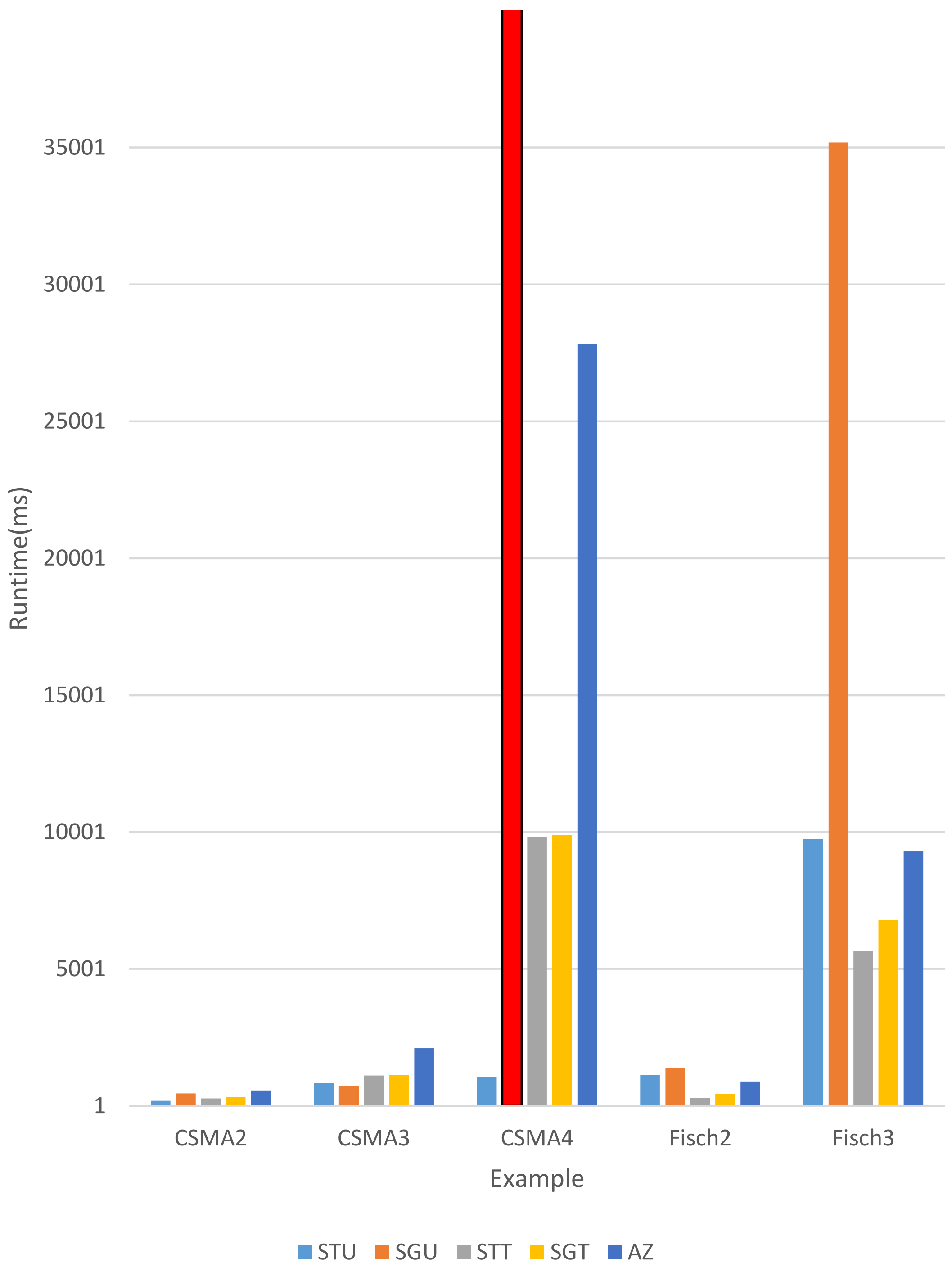


Figure 4.6 Results of measurements on Fischer and CSMA/CD protocols

- STT denotes state space-based refinement operating on the tree representation of the abstract zone graph where the precisions of the zones are based on the trace activity, and
- SGT denotes state space based refinement operating on the graph representation of the abstract zone graph where the precisions of the zones are based on the trace activity.

The algorithm denoted by SGU run out of memory when executed on the timed automaton representation of four stations communicating with CSMA/CD protocol – hence the red colour.

4.2.3 Evaluation

The performed measurements are not exhaustive, thus one should not come to far-reaching conclusions. However, there are some observations worth mentioning.

From the CSMA/CD examples it can be concluded that automaton-based refinement (algorithm AZ) does not scale well. The cause of this can be explained by considering automaton based refinement as a special case of state space-based refinement where the calculated precision assigns the same set of clock variables to each node, and that refines the complete graph not just a trace. Automaton based-refinement recovers so much of the hidden information that it prevents efficient model checking.

Out of the algorithms with state space-based refinement, the ones denoted by STT and SGT seem to perform better than the others. STT may be a bit more efficient, but their results are roughly the same. This is surprising, because the abstract state space representation is different in the two algorithms, and many of the techniques used in the algorithm are representation-dependent. It is interesting that the efficiency of two algorithms using different graph representations is almost the same.

Algorithm STU performs well on the CSMA/CD inputs, but this does not seem to be the case for the Fischer examples. However, it is worth mentioning that STU is the only algorithm that has been successfully ran on the Fischer protocol of four participants in the performed measurements. On the other hand, algorithm SGU hasn't performed well on either type of the test cases.

In conclusion, out of the presented algorithms STT seems to be the most efficient, and SGU seems to be the least efficient one. Measurements also suggest that state space based refinement may be generally more efficient than automaton based refinement.

Chapter 5

Conclusions

This chapter concludes the contributions of this paper and presents my future goals.

5.1 Contributions

I have developed a configurable framework for CEGAR-based reachability analysis of timed automata extended with discrete variables. The framework provides two different realizations of the CEGAR-loop: one, where the refinement is based on the automaton and one, where the state space is being abstracted and refined.

In the framework I have collected various techniques that can be used during the described realizations of the CEGAR-loop. My algorithmic contributions include

- a bounded model checker for reachability analysis of timed automata with discrete variables,
- a method for transforming execution traces of timed automata to SMT formulae,
- two representations of an abstract zone graph that can be calculated form the automaton, with operations for state space exploration and refinement, that are guaranteed to keep the graph an abstraction of the zone graph,
- two methods for calculating the precision to refine the zones on an execution trace in order to decide if it is feasible:
  - one, that is based on the *unsat core* function of SMT solvers and
  - one, that is based on the *activity* property of clock variables and
- a method for calculating the state space of a timed automaton with different precisions along the zones.

As a result the framework currently provides six different algorithms for reachability analysis of timed automata, but it is extensible.

I have implemented the presented framework in the theta framework and I have demonstrated the efficiency of the implemented algorithms with measurements.

5.2 Future work

The first and most obvious improvement option is to extend the framework with new modules, abstract zone graph representations and abstraction techniques, such as predicate abstraction [13]. I would also like to introduce algorithms that apply abstraction to the discrete variables as well as the clock variables and I would like to create algorithms, that combine different abstraction techniques, to eliminate spurious counterexamples in early phases of the verification.

I would also like to perform exhaustive benchmarking to see what combination of techniques is the most efficient for different kinds of timed automata and I would like to measure the performance of the algorithms on industrially relevant examples.

Acknowledgement

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