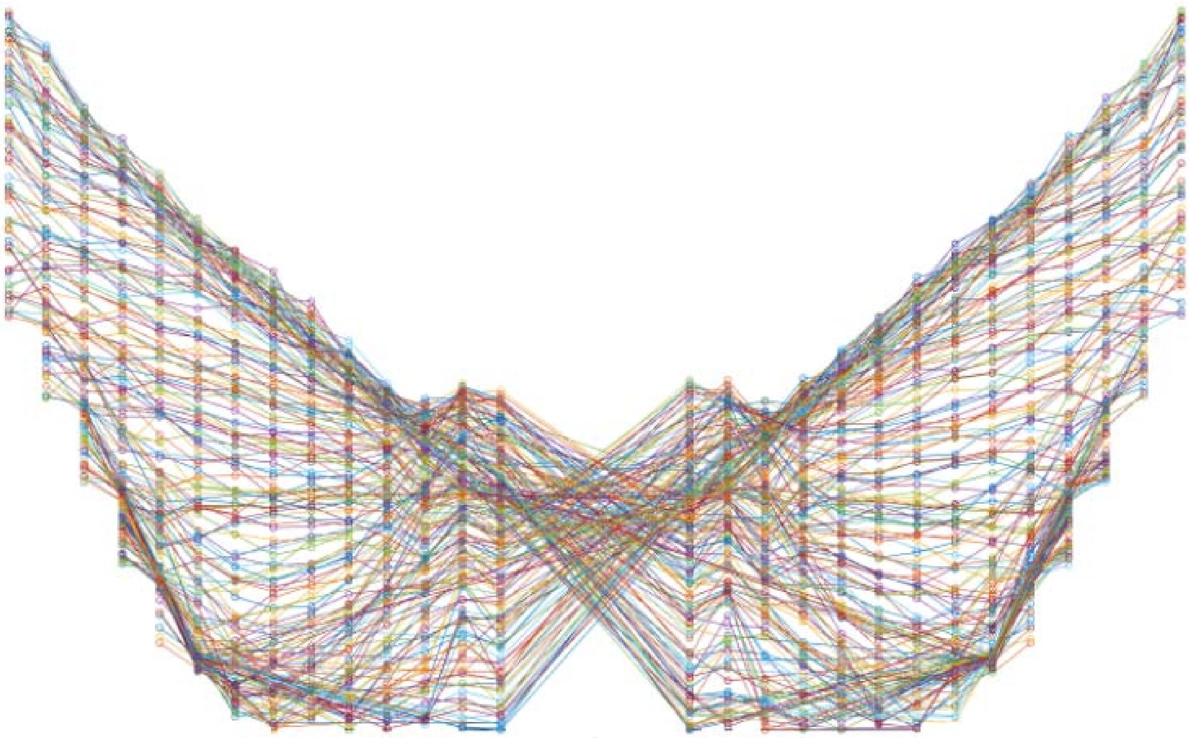
TDK Conference 2016

Experiments and numerical models for eigenfrequency identification of fluids



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Abstract

From the point of the structural reliability it is very important to well determine the loads and other effects acting. Generally, most of the possible effects are well approximated or at least they have a quasilinear impact on the structure. The problem is when we cannot forecast the nature of the effect or the connection between the action and the effect is highly nonlinear. We chose the eigenfrequency identification as our topic because we know that the same periodic load has a much larger effect on the structure if its frequency is close to the system's natural frequency than if it is not.

In this paper, we will introduce a special case of a water tower model to be able to compare the available analytical solutions with laboratory experiments and numerical models. The analytical solutions are based on "A szél dinamikus hatása az építményekre" (L. Kollár, 2004) [2]. The chosen numerical model is the so called Smoothed Particle Hydrodynamics (SPH) which can handle the problem of the free water surface in an appropriate way.

This paper is a sequel of our original work which is the prelude of this project. Originally we investigated in the laboratory experiments to see if we can accomplish an experiment on our own and we can compare our results with basic numerical models. The difference now is that we investigated in the post-processing part of the experiments and also we invented many new numerical models to examine specific phenomena.

1 Introduction

Working with solids is a well-known problem for civil engineers. There are plenty of experimental, analytical and numerical solutions. Many approximation and overestimation exists to guarantee the expected safety. The problem is much more complex if we must deal with fluids. Even in this case, there are two different options based on the relationship of the fluid and the structure: when the fluid acts on the structure (e.g. wind) and when the fluid takes part in the structure (e.g. water towers). In the first case, we should guarantee that the fluid will not cause periodical loads in such way that the frequency of the load is the same as the natural frequency of the structure. On the other hand, if the liquid is part of the structure, we still should guarantee that the external periodic loads' frequency – if any – will not concur the eigenfrequency of our system. So, the problem is: how can we determine the eigenfrequency of a structure containing liquids?

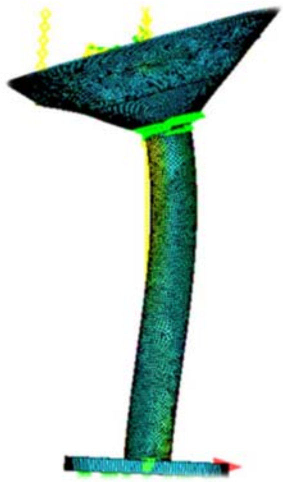


Figure 1 - Resonance of a water tower - numerical model

These kinds of problems are very complex and are lack of mathematically well deduced solutions. One possible resolving strategy is to determine the natural frequency of the parts and using the summation theorems determine the absolute eigenfrequency of the whole structure. In this paper this method will be followed. In this case, if we already know the natural frequency of the structure without the dynamic of the fluid, the final result will depend on the natural frequency of the liquid as well. It also means that the only unknown in general cases is the natural frequency of the fluid. For this reason, our goal is to determine the eigenfrequency of the fluid considering the boundary conditions prescribed by the structural parts, independently from other effects.

Once we have determined the behavior of the liquid, further calculations can be performed. Please note that the effect of foundation for example in case of towers shall be combined with the cumulated structure-fluid natural frequency.

In the following chapters, by eigenfrequency we mean the first mode of the natural frequencies. The analysis of further modes is so complicated that those are not under the scope of this paper.

To be able to handle these questions, the below discussed problem was modified. The whole analysis (mathematical, numerical and laboratorial) was performed on a scaled model. For simpler comparison, the analytical and numerical models were calibrated on the experiment's one with geometry shown in Figure 2.

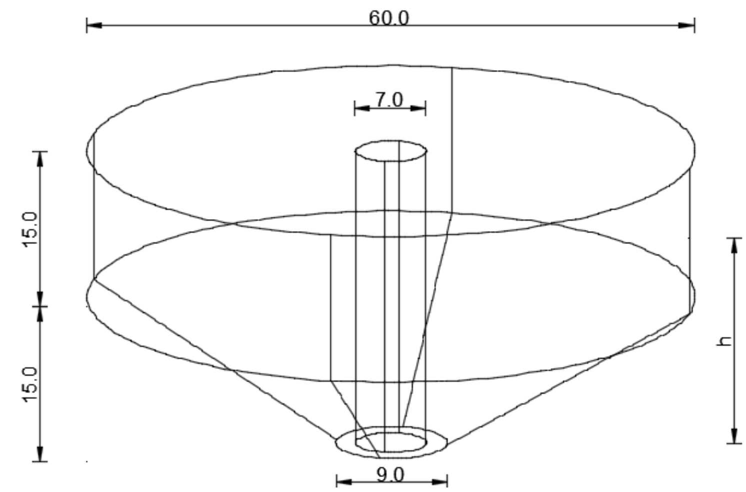


Figure 2 - Geometry of the model [cm]

The geometry has two parts: the lower part is a truncated cone with the minor diameter of 9 cm, major diameter of 60 cm and the height of 15 cm. The upper part is a joining cylinder with the diameter of 60 cm too. The geometry also contains an inner cylinder with the height of 30 cm and the diameter of 7 cm. This part was built to influence the fluid behavior according to the similarly constructed real-life water towers structure. The actual level of the water is denoted with "*h*".

Please note that since during the experiments, we could measure the water volume, all the results in this paper were considered in water volume and not in water level. Even if we use the term water-level, we considered the equivalent water-volume.

2 Analytical approximations

2.1 The rocket problem

The problem is known for long, although no applicable solution was invented yet. In the '60-s the NASA was working on a very serious problem: during rocket launches the shaking of the liquid fuel caused serious instabilities in the rocket. To better understand the phenomenon, they started a very thorough research. In 1966 H. Norman Abramson published a book based on the NASA research, named "The dynamic behavior of liquids in moving containers" [1]. Although many of the basic shapes were tested, it become clear that the slightest changes of the geometry can make the results discredited.

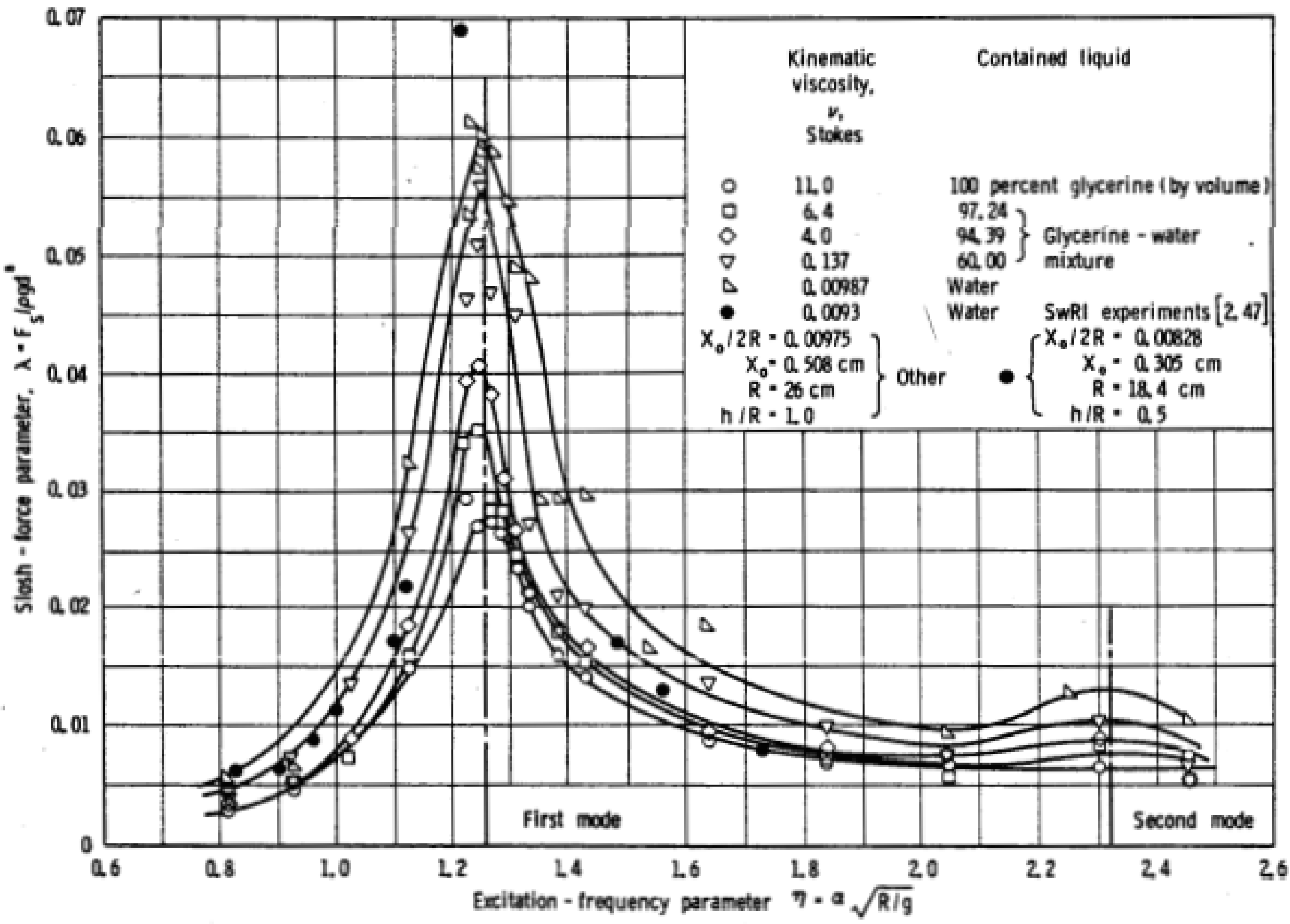


Figure 3 - NASA result - conical tank, viscosity is changed

Without further explanation, we would like to cite one of their results: The natural frequency of a liquid in a cone is [1]

$$\omega^2 * \frac{r_0}{g} = 1.84 * C_2^3$$

Where r_0 is geometrical parameter and $C_2{}^3$ is material parameter.

So, although in case of pure geometries these results can be used, in most civil engineering cases the structure itself is modified by other extensions, equipments or other important parts that these equations might not be used. For this reason, the approximation based on this method was neglected in our project.

2.2 Geometry based approximation

Each and every shape has its own characteristic, but of course formulas cannot be created for each and every situation individually. For this reason, a more universal solution shall be found. One possible approximation comes from Lajos Kollár [2]. This method is based on the touching sphere. Considering convex, upward open tank, imagine a sphere in such way that it touches the tank exactly there on the tank where the water level is (see Figure 4).

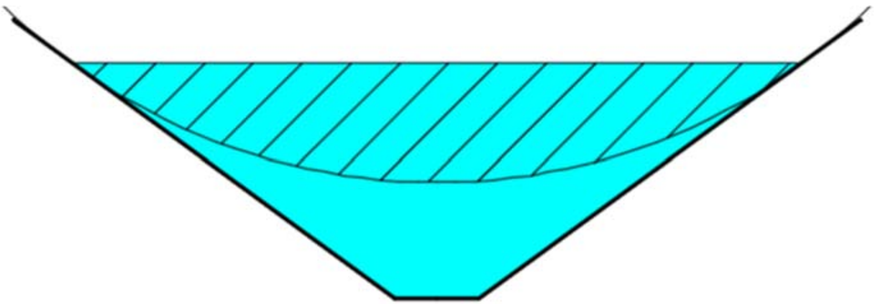


Figure 4 - Touching sphere

According to Kollár, the water consists of two parts: a moving and a not moving one. The movements of the particles around the wall of the structure, or farther from the free upper surface are obstructed so they almost fixed relative to the structure. As shown on Figure 5, the remaining part, which is excised by the touching sphere can move, so the natural frequency of the fluid exclusively depends only on this part, so in the next steps, only the moving part of the water is considered.

[1] Page 57. Equation 2.64

Figure 5 - Water movement in a bottle[2]

The natural frequency of the fluid in this case is:

$$f = \frac{1}{2\pi}\sqrt{\frac{M * g * h}{I}}$$

Where: *M* is the total mass of the considered water [kg]

g is gravitational acceleration [m/s^2]

h is the distance of the water's mass center from the axis of the rotation [m]

I is the inertia of the water to the axis of the rotation [kg*m^2]

Please note, that the axis of the rotation goes through the middle of the touching sphere.

As this method originally requires a convex tank, we should keep in mind that the middle column can make the particles slow down which could extend the periodic time which results a higher frequency. Even if we could consider the excluded part in the geometry, the above-mentioned idea may interfere with the analytical results!

[2] The shape of the mobilized liquid is not perfectly spherical, but please keep in mind that the theory is adaptable for liquids with small movements. In our case, even if the results were good enough, the shape of the mobilized liquid becomes distorted.

The tank has two parts: the cone and the cylinder. In the cone, the touching sphere could be easily drawn. Using AutoCAD all the necessary geometrical properties could be reached easily which lead to the below detailed results. On the other hand, the cylinder has one clear case only. If the touching sphere is too close to the joint of the cylinder and the cone, the sphere could not be placed fully in the structure because the cone is too small for it. This not only affects the geometrical calculations but furthermore destroys the possibility of the mobilization of the water. In this paper, we did not investigate in the analytical solution of the joint's environment. Aside this, the cylinder has the clear case: when it is high enough, the touching sphere could be placed fully in it. This means only one case since the radius of the cylinder has one parameter and the sphere is independent of any other parts. In this case the mobilized water is the lower half-sphere with the radius of the cylinder.

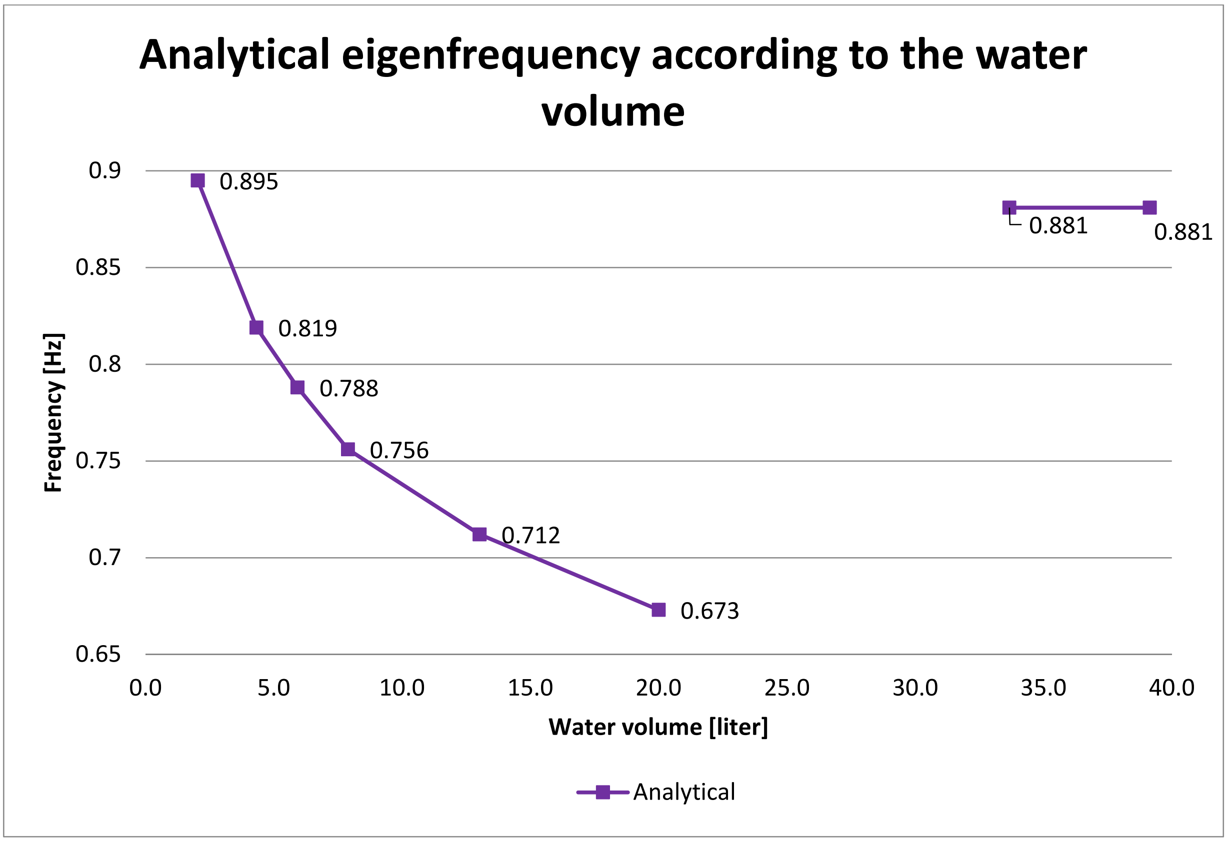


Figure 6 - Analytical results

Considering Figure 6, three very important conclusions can be drawn:

1. If the water touches only the cone, the higher water-level means the lower eigenfrequency (the higher periodic time). Considering the longer available movements for the liquid it makes sense.
2. Since the purely cylindric relies on constant parameters, this result is water-level independent. Once we reached this state, the curve reaches its plateau.
3. Since the water-level is a continuous property of the system and we know that every system in the nature has a natural frequency, we assume that the water-level – eigenfrequency diagram is also continuous.

3 Laboratory experiments

3.1 Structure

As there are too many questions in the characteristic of the fluid behavior, experimental results were needed. For this reason, we built a model water tower (see Figure 7) with the same geometry described above. The model was made of a 1 mm thick steel plate which wall was rigid enough to be considered as a fixed boundary. All the joints were welded.

Figure 7 - Water tower model

To assure steady periodical movements, we used a vibrating table. To guarantee that the structure's displacements are the same as the vibrating table's one, the structure was welded even to the table at some points as shown at Figure 8.

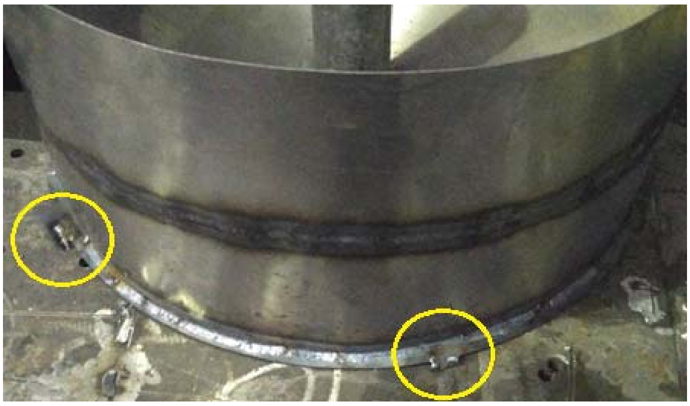


Figure 8 - Model-table connection

3.2 Loads

The vibrating table assured a steady sinusoidal horizontal displacement function. In each load case the amplitude of the displacements was constant but as we changed the frequency of the shaking, in order to reach a constant maximal acceleration from step to step, the amplitude had to be decreased.[3] This is also important in the numerical modelling.

For demonstrational purposes, we also created a so-called sweep load case, where the frequency (and so the amplitude) of the system was linearly changed in time slowly enough to let the system react. This is also useful when the natural frequency should be esteemed.

As our preliminary assumptions showed, we were looking for the natural frequency in the 0.60-1.50 Hz range. The lower part was not so important, but for saving time, it was useful. Not like in the upper part: if we considered too high frequencies, second or third modes might appear. As we stated earlier, that we are only interested now in the first modes, the upper limit should be chosen carefully. As we experienced, the given frequency range was satisfying during the tests.

3.3 Test cases

A test case consists of a given water level and a given excitation frequency. A test series consists of different excitation frequencies at the same water level. Please note that the amplitude is also changing but it depends only on the acceleration. For the sake of simplicity, during the experiments we did not measure the water level but we measured the total water volume. Depending on the geometry of the tank, the exchange from water level, to water volume or vice versa is unequivocal.

Please also note that if once we reached the natural frequency of the water, and the increased frequency showed a less moving water mass, we stopped the series and started a new one with a higher water level. We could do this because the behavior of the fluid at its natural frequency is so obviously different from other states that we could safely determine it.

[3] The higher frequency with the same displacements would mean higher velocities and higher accelerations which also means higher total energy. But higher energy rate would mean different state so the load cases would not be comparable.

We have done the tests given in Table 1:

Volume [L]	f.min [Hz]	f.max [Hz]	f.step [Hz]
2	0.7	0.96	0.01
4			
6			
8			
10			
12			
14			
16			
18			
20			
22			
24			
30			
34			

Table 1- Test cases

3.4 Problems

During the tests, we have faced some problems.

1. The early measurements were slightly false as we did not have a sophisticated measuring protocol.
2. At extreme low water levels, the water surface and depth was so little that the accelerometers had no space to move. This caused slightly false data, but the visual evaluation works much better in these cases.
3. The wires of the accelerometers have a certain stiffness which obstructs the accelerometers to move (Figure 9). This compromises the results.
4. The range of the acceleration was so low that it was difficult to set the vibrating table to the appropriate state. The noise of the controlling signal might be significant.
5. At high water level, some water was lost. The total loss at the end of the measurements was around 0.2 liter. This might not play a major role in the results.

Figure 9 - Accelerometers

3.5 Evaluation

Three investigation methods were used to determine the frequency level where the maximum displacement/movement of the water occurs. This frequency will be the eigenfrequency.

3.5.1 Visual (on-site) evaluation

As we wanted to save time on neglecting measurements which are not around the eigenfrequency, a fast and reliable method was needed. Obviously, the measurements were done with the accelerometers, but the results were not visible on real time, for this reason we had to determine it in another way. Fortunately, as it was mentioned above, the state of the fluid at the natural frequency is so different that it could be seen easily in most of the case. For this reason, we identified the approximate natural frequency of the fluid visually, mostly by checking the maximum displacements. Around this range (±0.02 Hz) we have done many tests with the accelerometer too. The results from this investigation are plotted on Figure 11.

3.5.2 Accelerometer based evaluation

The movement of the water was measured by five 3D accelerometers floating in the water. These accelerometers/inertial measurement units (IMUs) were connected to a microcontroller which collected the data, and transformed the accelerations to a global (Earth-referenced) coordinate system. This transformation was important as the alignment of the sensors' axes is not maintainable when floating in a fluid. From this setup, we collected data series for each accelerometer with 100 Hz sampling rate. The global coordinate system's axes were parallel with the vertical direction (*z*), North (*x*) and East (*y*). The acceleration was plotted as a function of time. Total measurement duration was 10 seconds.

In Figure 10. the accelerations from the accelerometer 1 and 3 are plotted, in z directions (water level is at 14 liters). The plot shows 6 different lines for the 6 different frequencies examined (0.76, 0.77, 0.78…0.81 Hz). In case of this water level according to the visual examination (measurement method 1) the 0.79 Hz showed the maximum displacement. On the figure the purple line is related to this frequency, but it is also visible that the yellow line (0.78 Hz) and green line (0.80 Hz) as similar acceleration level. This means that according to the acceleration we cannot be confident with the exact result at that time. On the other hand, it is clearly visible that the blue line (0.76 Hz), the red line (0.77 Hz) and the light blue line (0.81 Hz) cannot be the eigenfrequency of the water. For these reasons, we need further investigation, but at that point we concluded that the maximum amplitude (eigenfrequency of the water) is between 0.78 and 0.80 Hz. From these maximum accelerations, the average values were collected and they are shown on Figure 11.

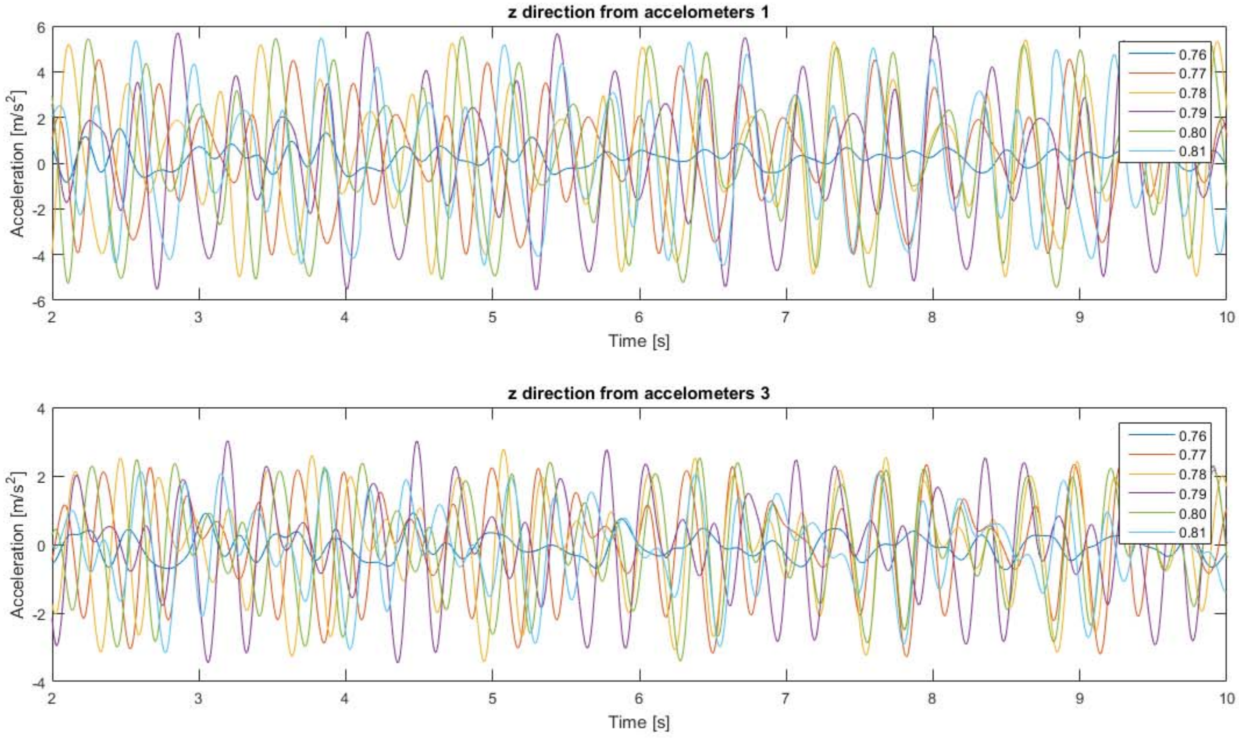


Figure 10 – Accelerometer results

All the measurements for different water level from 2 to 34 liters were plotted on the same way as the case with 14 liters mentioned before. The results figures attached to the appendix.

A low-pass filter was used with 5 Hz, to filter out the movements which also appeared on the measurement, but in our case, it was irrelevant.

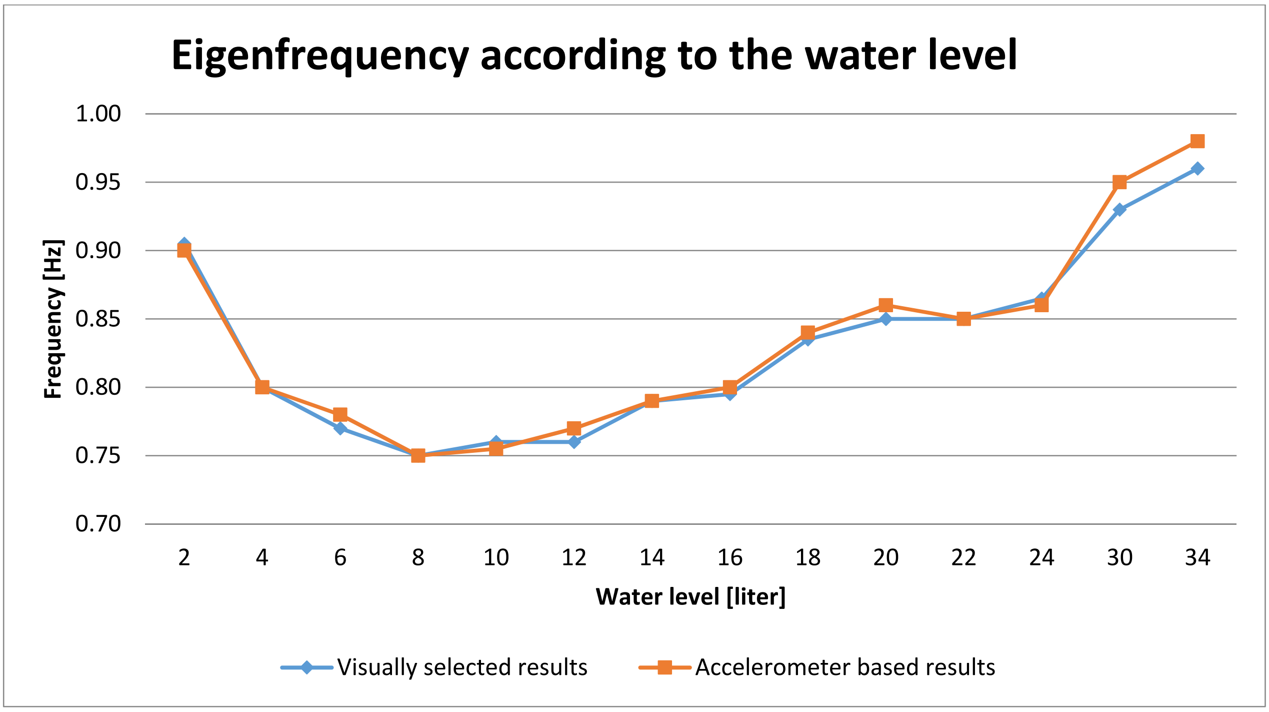


Figure 11 – Experimental based eigenfrequency according to the water level from two evaluations

3.5.3 Operational Modal Analysis

Operational Modal Analysis has been carried out to get more precise results from the accelerations. These evaluations were calculated by using MATLAB [4].

Operational Modal Analysis has been used to determine the mode shapes of the water in order to evaluate at which frequency level is the maximum displacement. We used a non-parametric technique based Frequency Domain Decomposition (FDD). The analysis technique allows determining the Resonance Frequency and Mode Shapes by using only the response of the structure.

First the Fast Fourier Transformation (FFT) was performed on the raw data to obtain the Power Spectral Density Matrices which contain frequency information. With the FDD technique the modes can be estimated by using a Singular Value Decomposition (SVD) of each of the data sets which were measured. The decomposition corresponds to a Single Degree of Freedom (SDOF) identification of the system for each singular value.

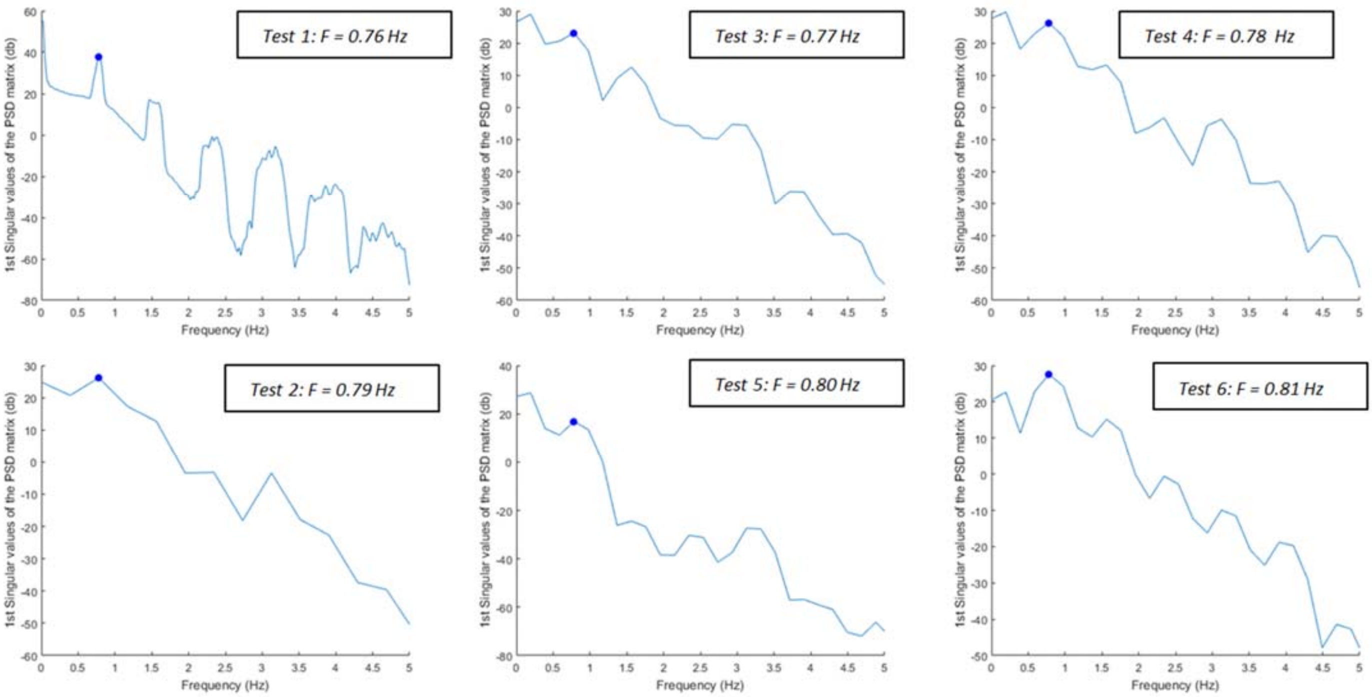


Figure 12 - Singular Value Decomposition for the measurement in case of 14-liter water level

Figure 12 shows the result of the SVD for five measurements of the same 14-liter water level. The peak peaking was done by clicking to the first highest point which was on the previously assumed interesting range. The peaked peak is marked with a blue point. We were just interested in the first mode shapes. On Figure 12 a sketch of the tank is visible with place of the accelerometers and the plotted mode shapes. On Figure 13 and Figure 14 the mode shapes were plotted from 0.76 Hz – 0.81 Hz. The frequency which cannot be the first eigenfrequencies were signed with a red 'x' as they are coupled with other mode shapes (0.76, 0.80, 0.81 Hz) it can see as their mode shapes does not describe a clear first mode. The line which ends 'the lowest' shows that its displacement is the highest. This is the red line with 0.77 Hz. According to the mode shape plotting this is the frequency which is related to the eigenfrequency. On the right top corner all the line ends at the value one, as the modes shapes are normalized to the maximum displacement. The values of the water level (z-axis) are not the real values as from a modal analysis just the shape can be determined. The distance between the accelerometers can differ, but the order is fix.

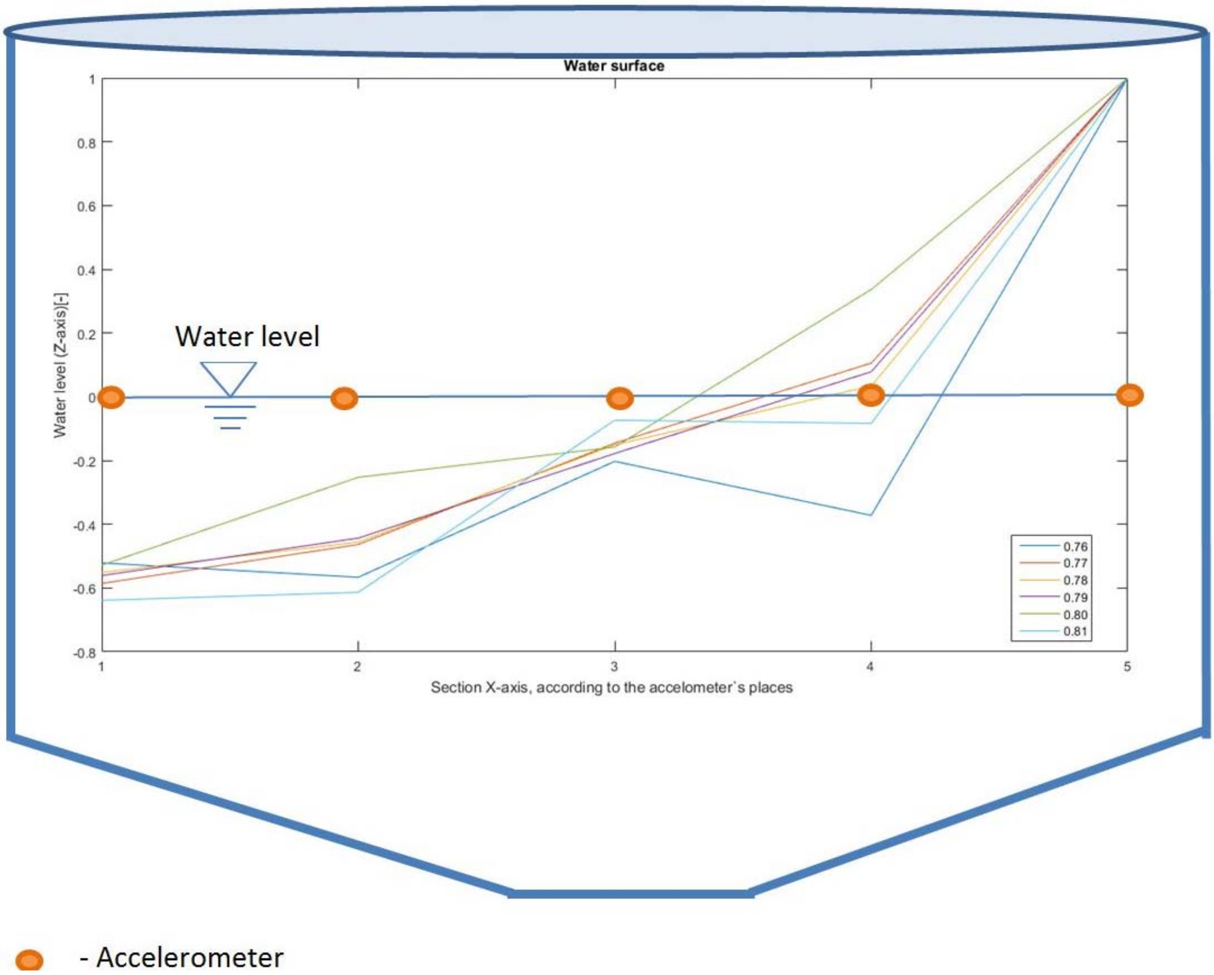


Figure 13 – Schematic view of the water tank, accelerometers and mode shapes

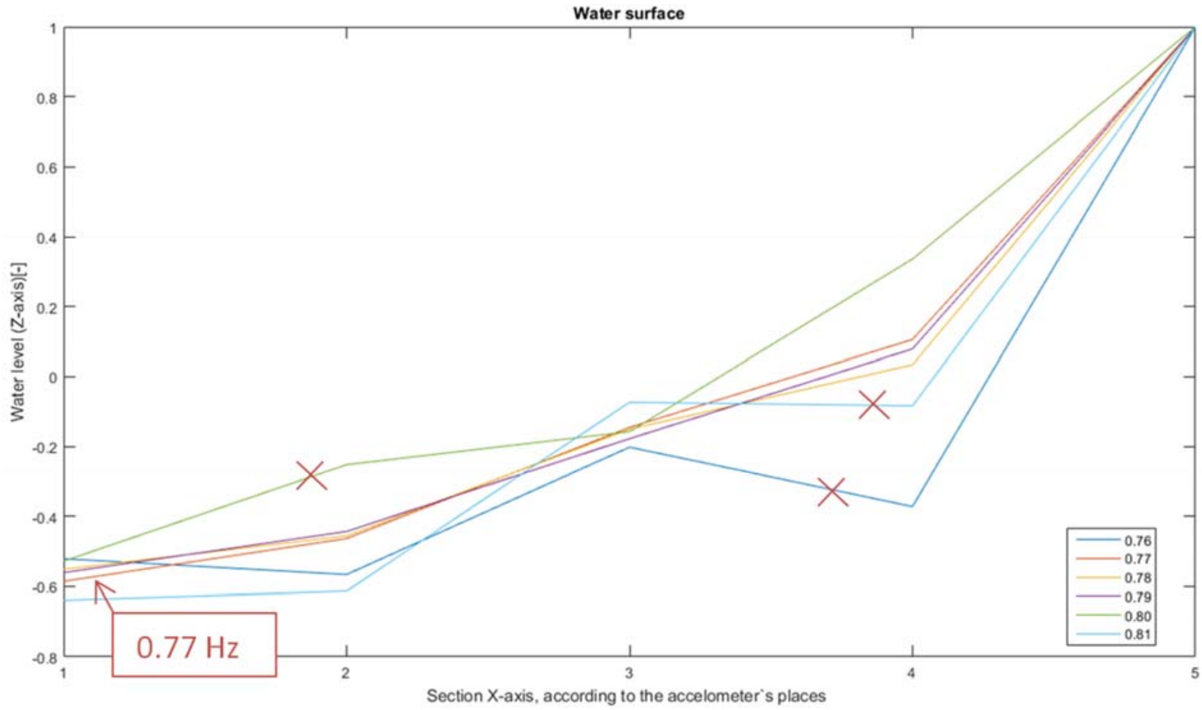


Figure 14 – Mode shapes are plotted at 14 liter water level

These mode shape plotting and analysis was performed at several levels and compared with the method 1. and 2. The comparison is shown below on Figure 15. It can be concluded, that even with this method the accuracy was the same as before. Figures are shown on the appendix.

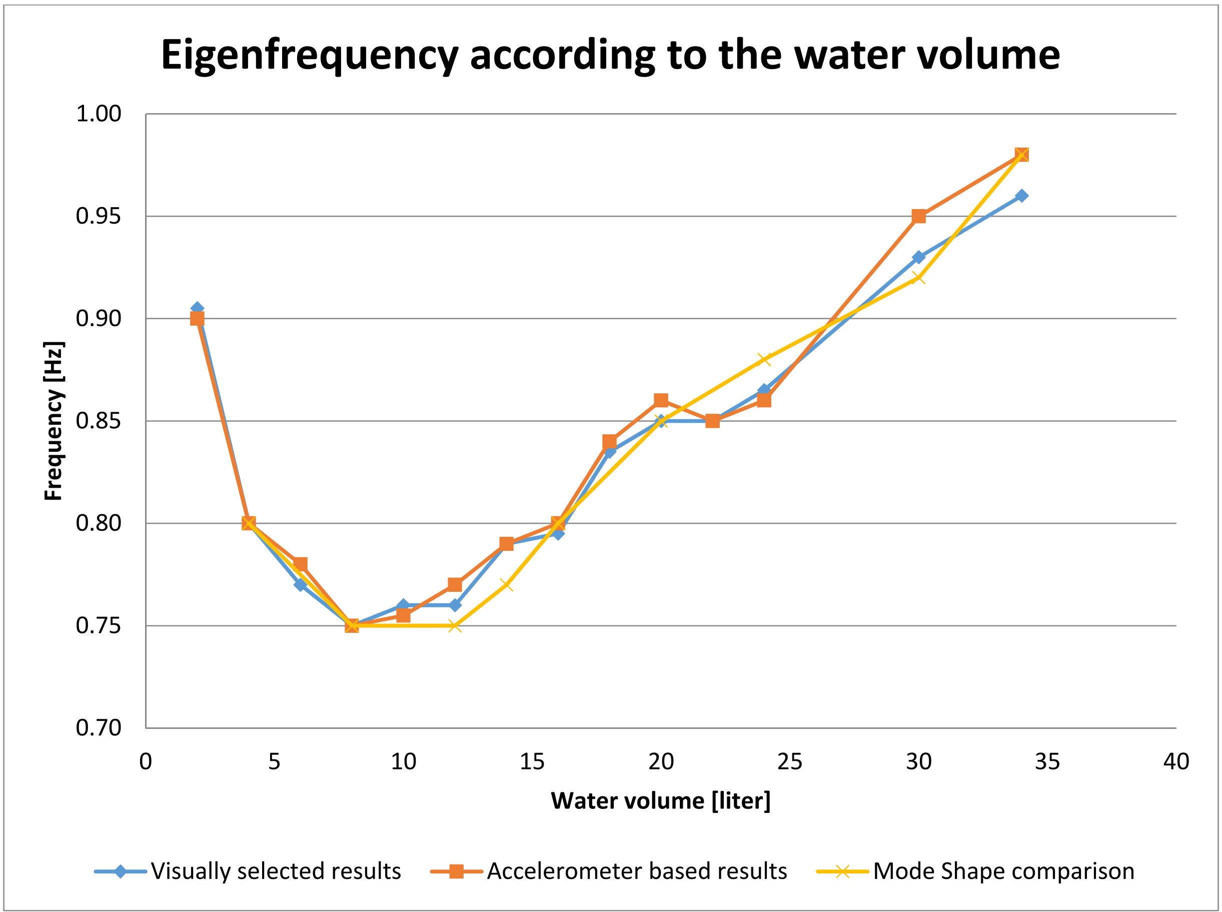


Figure 15 - Experimental based eigenfrequency according to the water level from three evaluations

The diagram on Figure 15 has two characteristic tendencies. The first is around 2-8 liter where the eigenfrequency is decreasing. This was also assumed that in case of more water the eigenfrequency will be smaller. The second is the increasing tendency after 8 liter. This is the water level, where the moving water reaches the vertical wall of the tank, from this reason the route of the water's particle become shorter. If the route is shorter the time become shorter also, such as the frequency is the reciprocal of the time, the frequency needs to increase after the point when the moving water level reaches the vertical wall. From similar reasons a plateau was expected after 20-liter water level. This is the level when the static water’s free water surfaces reach the vertical wall. In this experiment this plateau cannot be executed as the moving water reached ending of the wall and was leaking. For this reason, higher water level cannot be measured.

As a proof of the existence of the plateau a simple experiment was carried out with a glass of water. To make the water movement visible small lemon fiber was added and assumed that they have the same movement as the water. This experiment showed that in case of vertical wall the water seems to have a same moving behavior independently from the water level.

4 Numerical model

In order to create a general solution, a numerical model should be built. When we speak about computer based solutions, we have plenty of options. Generally, the most popular numerical solutions are the Finite Element Method (FEM), Finite Volume Method (FVM), Finite Strip Method (FSM), the Discrete Element Method (DEM) and of course the fluid mechanical solutions. As the problem itself is very special, it is our task to choose the most applicable solution.

4.1 Problems

Fluid movements are very complex. For example, it is clearly nonlinear. In this case the classical linear solutions (linear FEM) cannot be used. Nonlinear FEM at very large displacements might be used, but as the water particles are mixed, the connections between them become hectic so all the FEM solutions should be excluded.

A possible solution can be a DEM model. Although it might be mathematically an option, in order to reliably model the fluid, much more particles might be needed than in an average model we could use nowadays. The problem here is the runtime and the resource consumption.

Classical fluid mechanical solvers also can be used for this problem, but please note that most of them are built for closed systems, where there is no free air surface. Although it can be modelled, it is quite complicated.

For this reason, we chose a not so well-known solution: the Smoothed Particle Hydrodynamics method (SPH).

4.2 SPH in general

”*Smoothed particle hydrodynamics (SPH) is a method for obtaining approximate numerical solutions of the equations of fluid dynamics by replacing the fluid with a set of particles. For the mathematician, the particles are just interpolation points from which properties of the fluid can* *be calculated. For the physicist, the SPH particles are material particles which can be treated like any other particle system.*" [4] [3]

From our point of view, the SPH method is a particle based discrete element method like solution. In our case the big differences to a general DEM model are the following:

- The particles size is always the same
- The particles behave like the spherical ones in DEM (since they are point like)
- All the material properties of the particles are the same
- The walls are perfectly rigid but the particles can penetrate to their influence zone
- The results are not interpreted only on the particles but also between them

These simplifications help us to reach a much more time saving solution.

4.3 SPH model of the tank

As this method is not widely used, we applied a software what was written by Balázs Tóth. As the code is not commercial and not published yet, we got a preset version of it. It means that there was a set of variables that could be change beyond the geometry and the loads.

The modeling of the system happened very similarly to the laboratory experiment. Using the same geometry, different water levels were applied. At each and every water level, many loads were used. Basically one series (one water level) means 4-8 tests. In a series in each variant, only the load (the excitation frequency) was changed.

4.3.1 Details

The following important parameters were preset:

- Δt_{def} = 0.01 ms
- Δt_{min} = 0.001 ms
- $N_{particles,max}$ = 3 000 000
- ρ_{water} = 1000 kg/m^3

The following parameters were configured in each numerical model:

[4] J J Monaghan: Smoothed Particle Hydrodynamics - Monas University, Australia, 2005. – Chapter 1.

- s_r = 10 – 22 mm (particle's range. Analogue to element size in FEM) (default: 18 mm)
- T_{tot} = 3.4 – 20.0 s (total modelled time)
- t_{wall} = 18 – 22 mm (wall thickness) (default: 18 mm)
- h = 7 – 20 cm (water level)

In order to guarantee numerical stability, at some extremities (very low or high water level; very high frequency) the wall thickness should be increased or the particle size should be decreased. T_{tot} is set in each case in such way that the model runtime involves 3 full periods of excitation. At extreme low water level (h=7cm) it was increased to 7 periods.

Even this numerical solution is quite time consuming. Using a desktop PC with middle-ranged hardware, including SSD, the general unit runtime was the following:

$$0.5\,\frac{min}{liter * modelsec}$$

So, a 20-liter model, considering T_{tot}=4 sec is around 40 min. This model contains more than 1 000 000 elements.

4.3.2 Geometry

The geometry shown on Figure 16 is just the same as earlier. The wall thickness was considered in such a way, that the distance midline of the plate from the inner part was increased with the half of the thickness to take the exact same amount of water as we considered earlier.

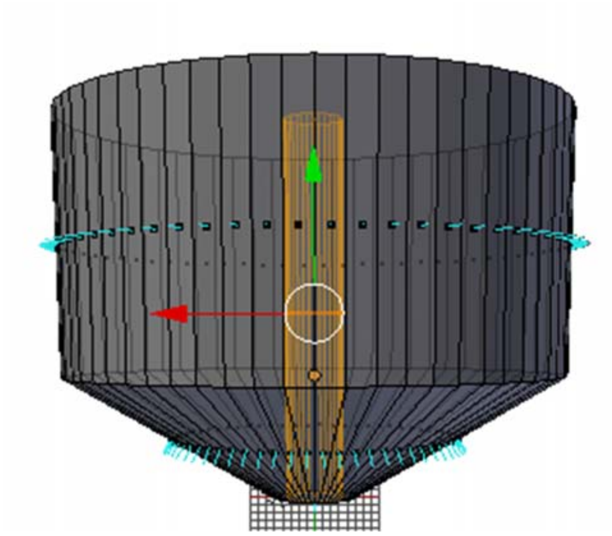


Figure 16 - Tank with its normal and reference coordinate grid

4.3.3 Fluid

Only one material was set in the model (the wall is considered as boundary condition). The parameters of the liquid are set in order to model clear water, detailed above. The particle size may vary on the other parameters in order to reach numerical stability.

4.3.4 Boundary conditions

The wall is totally rigid and fixed in space. No displacement is applied on any part of it. The wall has an influence zone ($\pm t_{wall}/2$) in where a particle penetrates, forces start to act on the particle. The surface of the water is free, but far away, the tank is closed to ensure mass conservation without significantly changing the numerical results.

4.3.5 Load

As the boundary condition is a fixed wall, moving it is not allowed, as it was solved in the laboratory experiment. A similar method was applied, namely the acceleration field was excited in a sinusoidal way where the vertical acceleration is constant (9.81 m/s^2) and the horizontal acceleration varies in the same way as varied during the experiments. The two methods are comparable if and only when the maximal displacements of the particles occur at natural frequency of the system. Since we have no better option, this approximation shall be applied.

4.4 Evaluation

Since the program is not commercial and under development, we could not use it as comfortably as a commercial one. The problem is that we could not use post processing features. The simplest thing what is reachable is the displaced shape of the structure in time. Since we could not define any value yet to describe the clutter of the system we chose the maximal displacements to find the critical parameters of the system at first.

Projecting the results on each other at one series, it is easy to choose the largest displacements. Here it is very important to use the right accelerations, as it was mentioned earlier. If the amplitudes would not decrease, the energy and the maximal displacements of the system would get higher and higher. A result might be considered as a local maximum only if it is not the lowest or highest considered value, of course. If it is not true, further computations should be done.

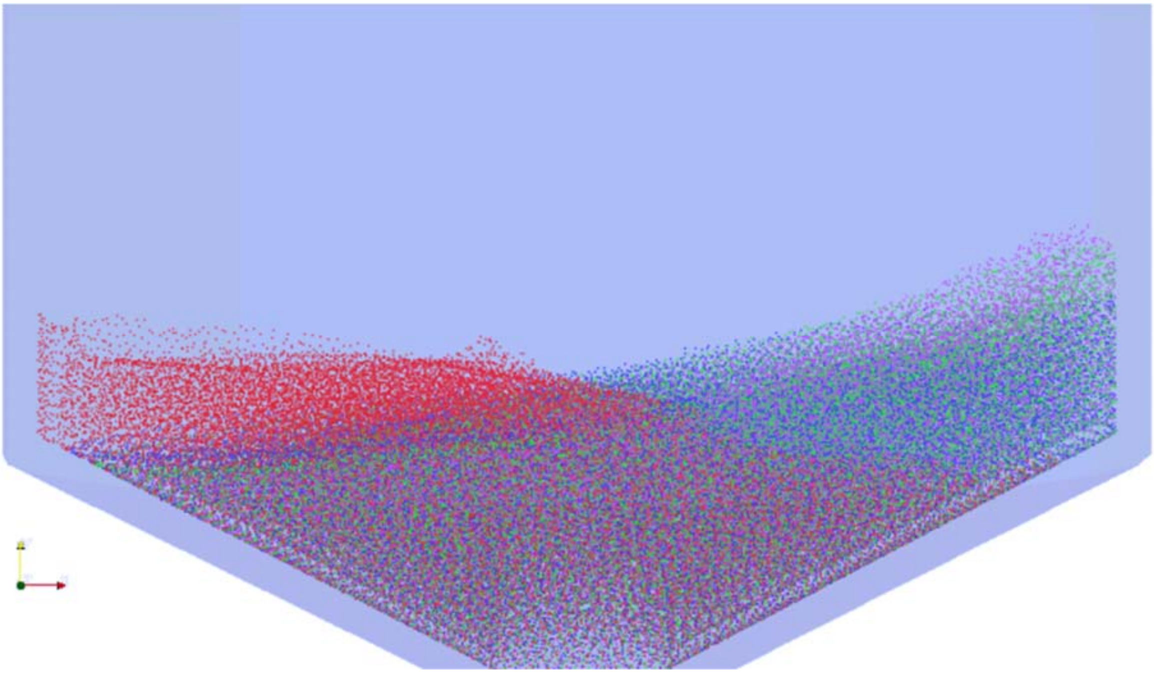


Figure 17 – Displacements

Please note, that at Figure 17 each color is a different model, projected on each other for analyzing the series. This method is very similar to the optical method at the experiment.

4.5 Examination of the particle size

Particle size means the distances between the middle of the particle, as one particle is concentrated into the middle point of a sphere. The examination of the particle size is an identical process as a mesh convergence analysis in a FEM.

In order to use an 'optimal' particle distance, 5 simulations were run for 5 different SR each in the same water level (13 liter) for 3 different frequencies. Optimal means for us here as the running time is acceptable and the result shows good convergence.

The results were summed up in the Table 2 - Particle size evaluation. The results in the table show that the distance needs to be between 14-18 mm, as in this range the average maximum displacement always occur at 0.78 Hz, where was expected from the previous evaluations. This means between 14- 18 mm we are constantly getting the average maximum at 0.78 Hz. In case of 12 mm high value occurred at 0.74 Hz, this can comes from the fact that the size of the elements are small, and they might have irregular movement. From this reason we recommend that 2-3 distances should be investigated and compare the result from them.

Frequency [Hz]	Particle size [mm]	Max displacement [m]	Average displacement [m]
0.74	12	0.205	0.1647
0.76		0.204	0.1628
0.78		0.202	0.1645
0.74	14	0.205	0.1617
0.76		0.200	0.1628
0.78		0.210	0.1630
0.74	16	0.196	0.1509
0.76		0.198	0.1510
0.78		0.197	0.1522
0.74	18	0.193	0.1463
0.76		0.192	0.1462
0.78		0.194	0.1463
0.74	20	0.182	0.1395
0.76		0.180	0.1397
0.78		0.179	0.1402
0.74	22	0.180	0.1673
0.76		0.179	0.1672
0.78		0.175	0.1656

Table 2 - Particle size evaluation

On Figure 18 a section of the moving water level was plotted in case of excitation 0.74 Hz with 14 mm partial distance. On the plot the Y coordinate refers to the vertical axis shows the displacement of the water, the X is the direction of the excitation. The average maximum displacement was calculated from the points in the box on the left.

For all the cases, which are mentioned in the table above a figure was prepared, are shown in the appendix.

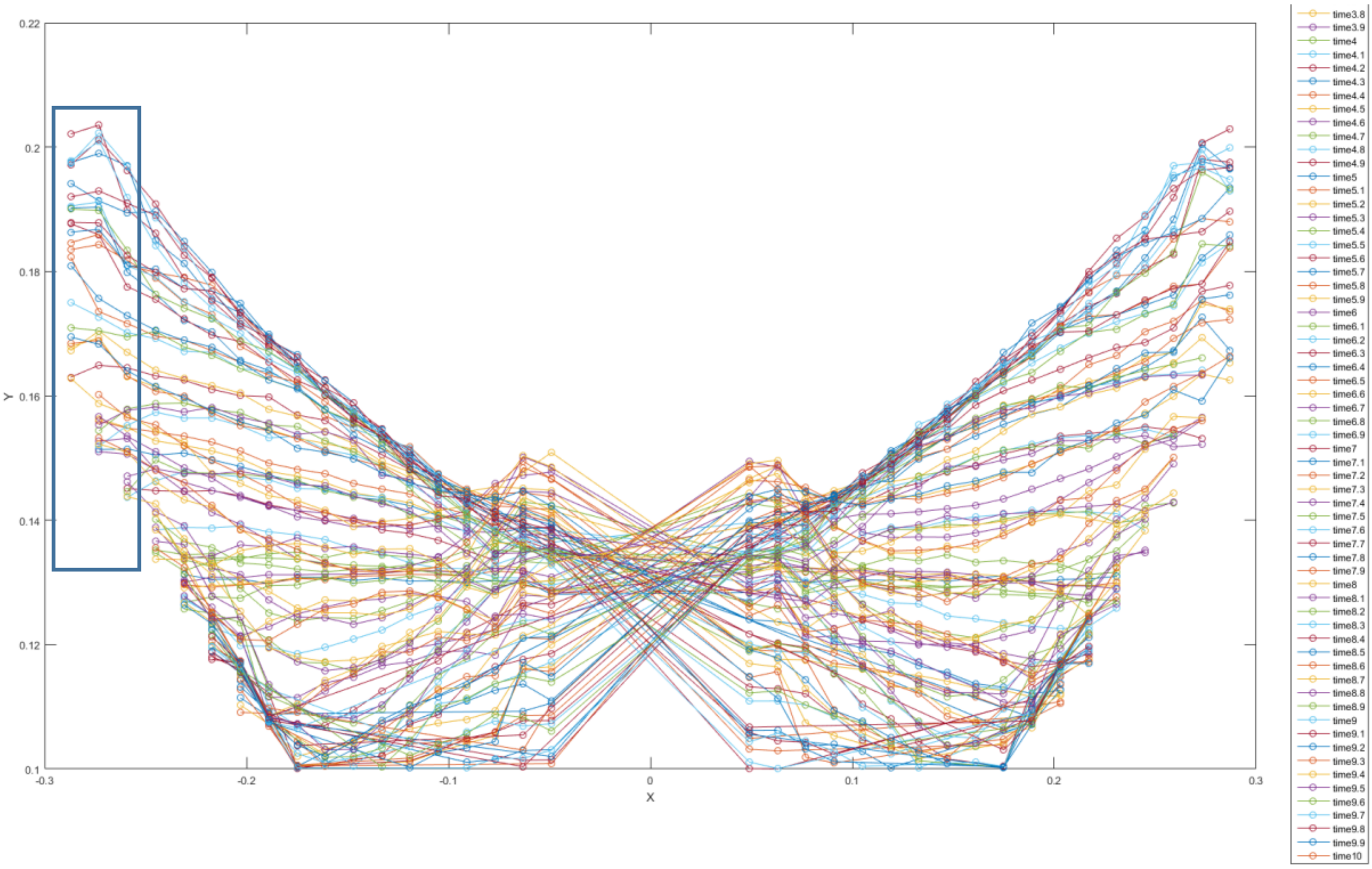


Figure 18 - Water level with 0.74 Hz excitation, Sr = 14 mm

5 Comparison of the results

Considering the results, the following diagram can be compiled:

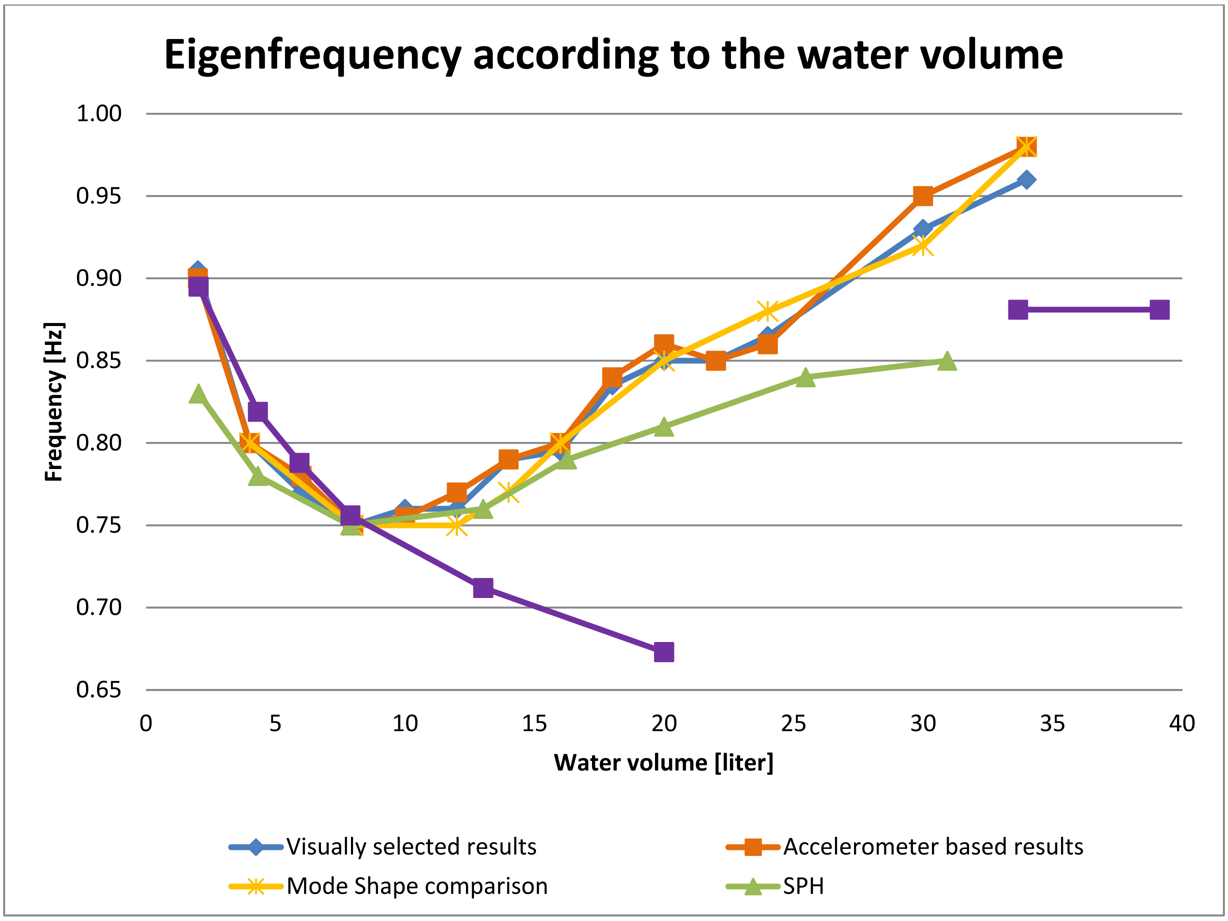


Figure 19 - Summarization of the eigenfrequencies

First, consider the hand calculations with the numerical models. The approximation given by Kollár Can be cut in three parts. First, the clearly conical describes the state where the touching sphere takes place in the cone and during the movements the water does not exceed the cone. In this part, the analytical results well-fit the experimental data, noting that we expected slightly higher analytical results considering chapter 2.2 – conclusion 1. In the second part, where during the movements the water touches the cylinder, the results tend to differ. While the laboratory results show a higher frequency according to the shortened path of the particles, the analytical solution cannot take into account this effect. The third case, the clearly cylindrical stage could not be reached during the tests.

Taking in account the SPH results, we could state that the plateau-effect seems to be close to the maximum modelled water-level. Also, the numerical results have a clear descending-ascending-plateau characteristic which is analogue to the analytical assumptions, but during the tests we cannot even get close to the maximum of the eigenfrequency. To make sure that in these high-water-level cases the first mode were found, further tests should be undertaken.

Notes:

- The decreasing-increasing characteristic of the system also exists in the numerical model, not only in the experimental results. In the SPH model, the same scene appeared: the water hit the wall so it turned back earlier, shortening the periodic time which increases the frequency.
- We did not care with the roughness of the wall's surface. Maybe at very low water levels it has a big effect on the fluid.
- Although the laboratory results are not monotone in some range, the numerical results are smooth. We might think that the imperfections of the model tank or the measurement errors caused this kind of uncertainty.
- At extreme low water level, the numerical and the measured data shows big differences. Maybe the effect of measurement errors or geometrical uncertainties play a big role in it. On the other hand, the particle size might be decreased in the SPH model.
- The SPH results are always lower than the measured data. The calibration of the model might be useful at least in such way that we could set two different numerical models to calculate a possible minimal-maximal natural frequency range of the fluid.

6 Summary

The natural frequency determination of fluids is a very complex problem. To be able to handle this question, we have done laboratory experiments in parallel with building an SPH based numerical model. Considering the 7-19 liter water volume range we have done 14 series of tests and 12+1 series of numerical calculations. As we can see, the results have well-fitting and not-well-fitting ranges too. Measurement errors, numerical errors and imperfections can cause these differences.

6.1 Further steps

- The main goal would be to reach a better fitting curve. This can be reached by the repetition of some tests and the calibration of the model too. To ensure the validity of the results, both of them might be needed, based on our experiences.
- Carry out a three-dimensional analysis of the water surface.
- To develop the evaluation of the data and to be able to describe and compare the models, post processing features might be needed in order to define the clutter of the system.
- On the other hand, do not forget that the loads during the experiment and in the numerical model are slightly different. Further investigations might be needed to examine the effect of this difference.
- Once we have validated our model with the experiments, other shapes and cases might be considered too, for example the NASA tests. [1]
- Considering all our results, we think that the SPH method in general is applicable for the determination of the eigenfrequency of fluids but it is even more important to validate of the models than in other numerical solutions.

Literature

[1] **H. Norman Abramson:** The dynamic behavior of liquids in moving containers – Washington D.C., 1966.

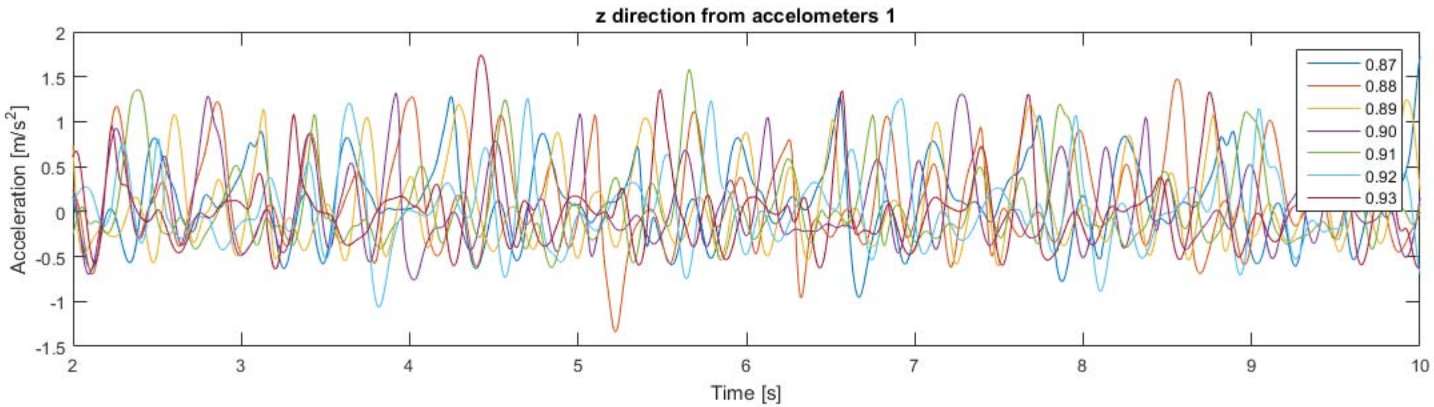
[2] **Lajos Kollár:** „A szél dinamikus hatása az építményekre" - Budapest, 2004. [Hungarian]

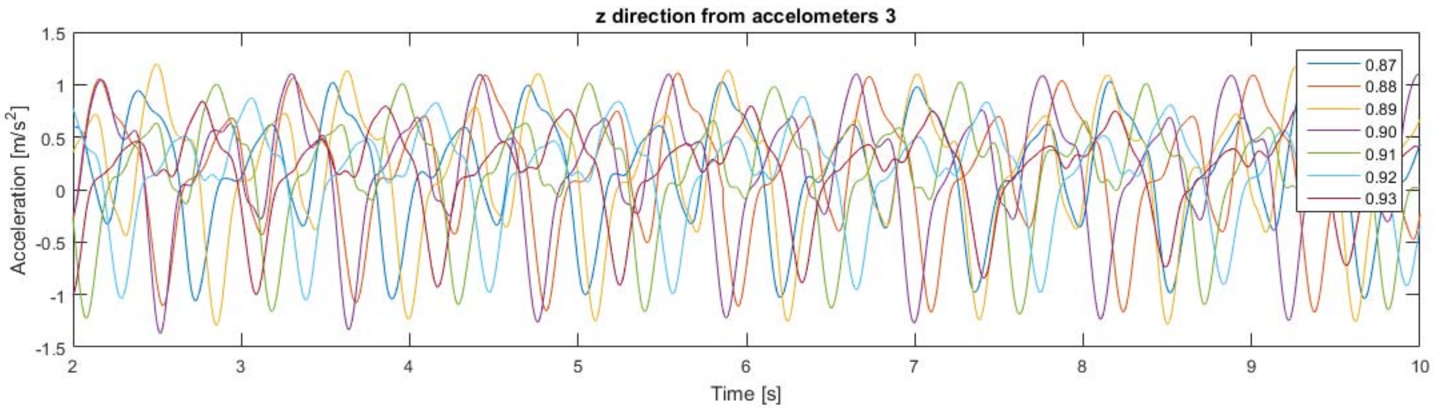
[3] **J. J. Monaghan:** Smoothed Particle Hydrodynamics - Monas University, Australia, 2005.

[4] **Mohammad Farshchin**: Frequency Domain Decomposition (FDD) https://goo.gl/1c4xQi

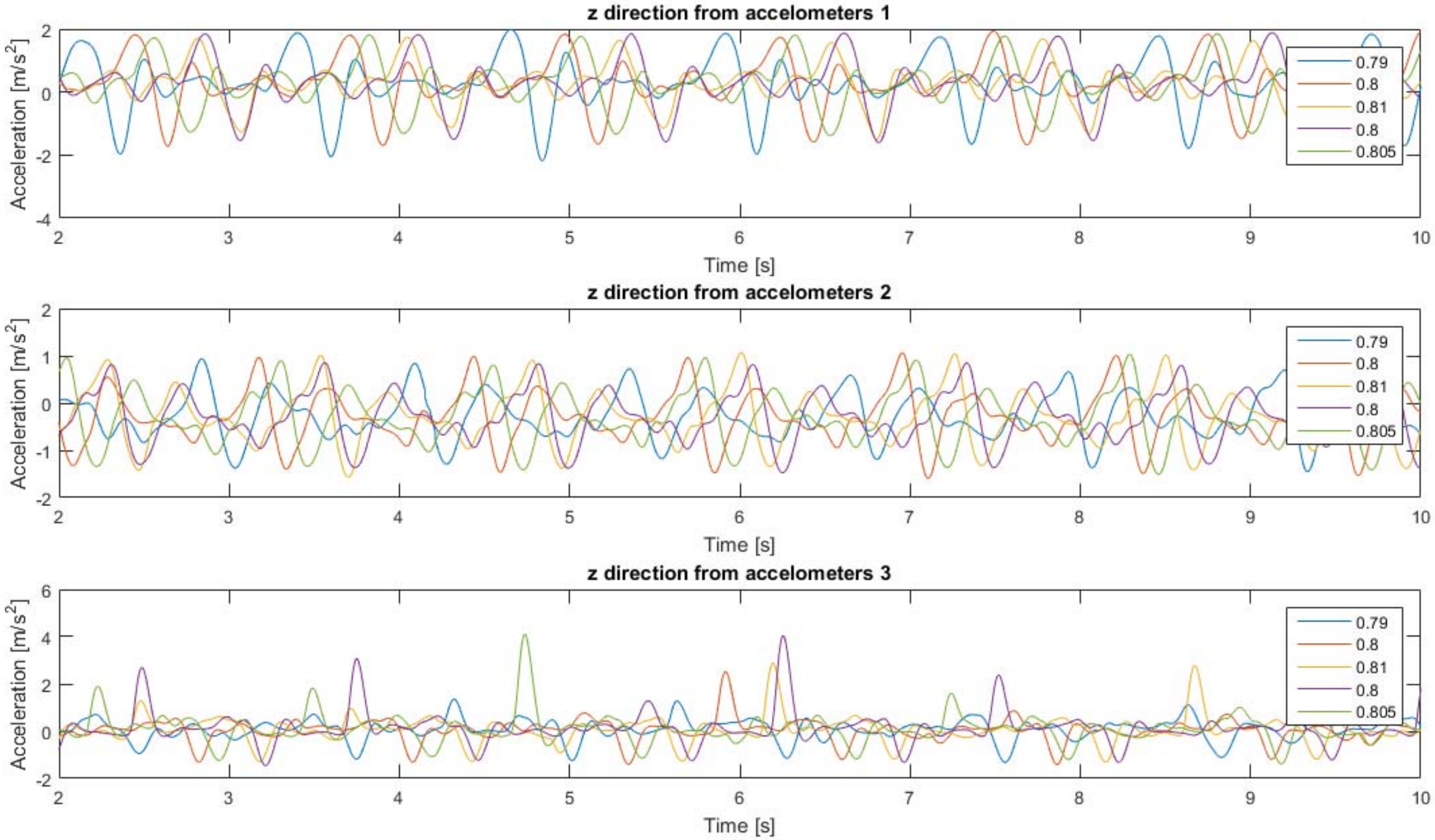
[5] **K. Ficker, A. Nagy, D. Szalay, M. Szedlák**: Experiments and numerical models for eigenfrequency identification of fluids – BME, Budapest, 2016.

Appendix

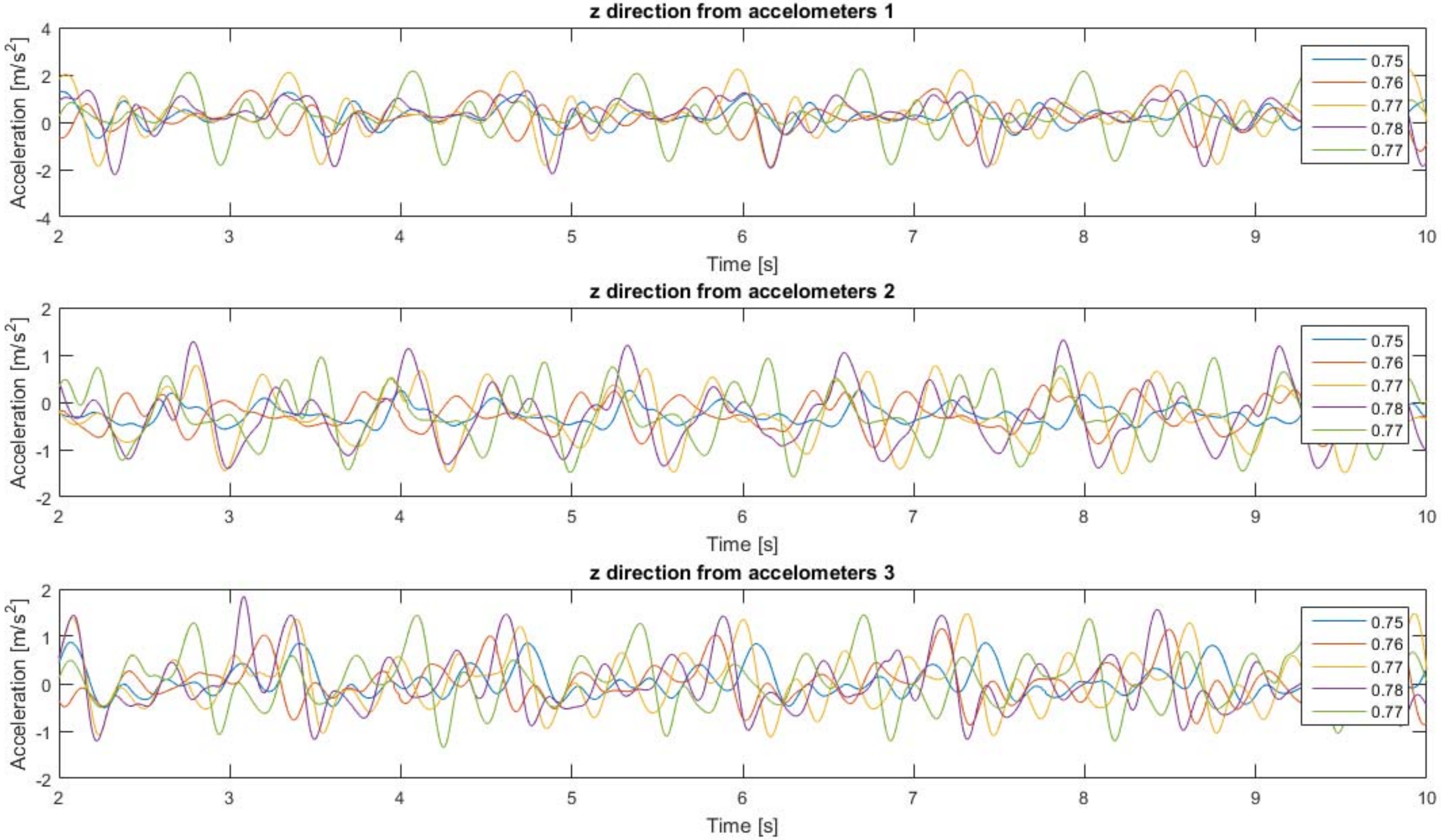

z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.87
0.88
0.89
0.90
0.91
0.92
0.93


z direction from accelometers 3
Acceleration [m/s^2]
Time [s]
0.87
0.88
0.89
0.90
0.91
0.92
0.93

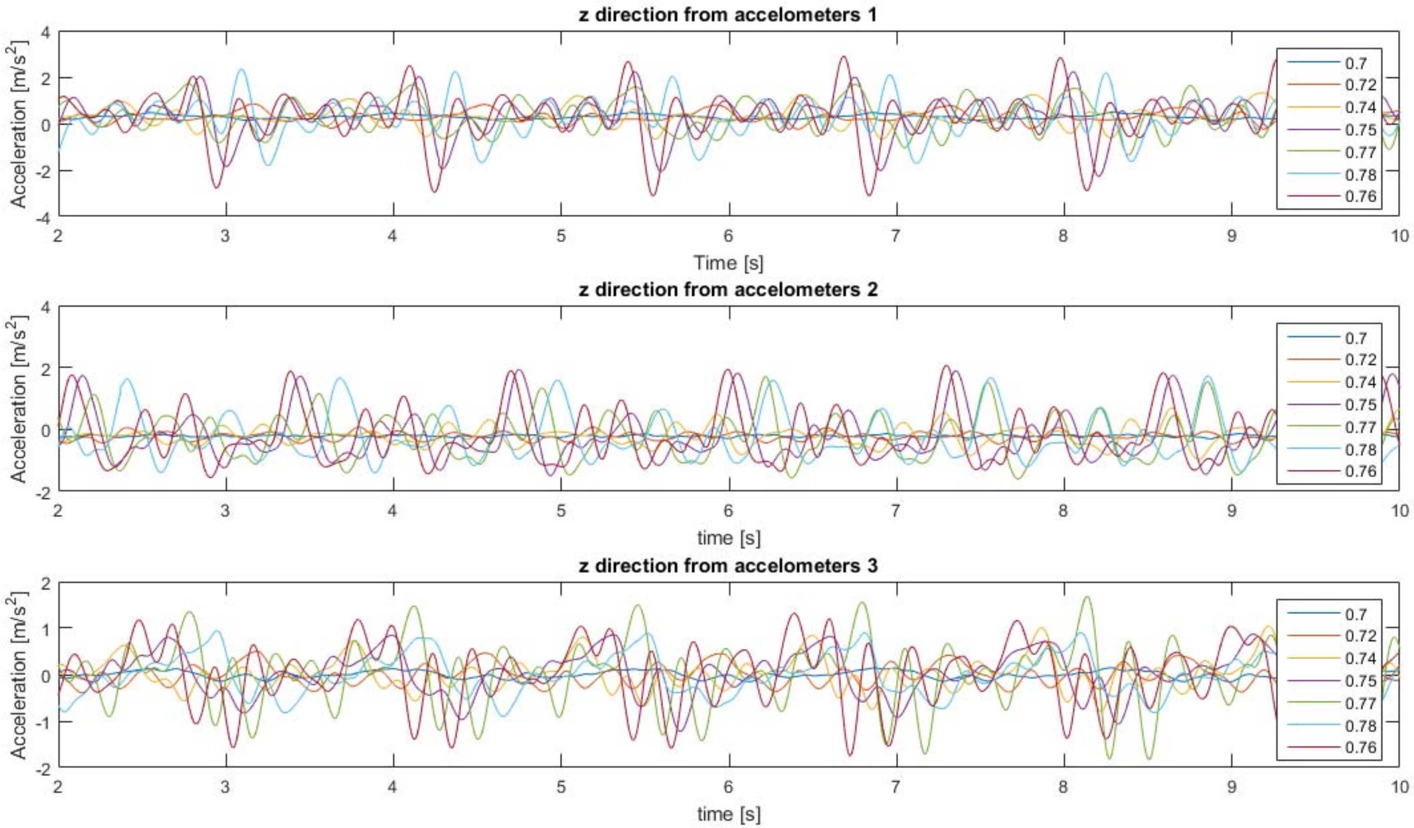
1 - 2 liter


z direction from accelometers 1
Acceleration [m/s²]
Time [s]
0.79
0.8
0.81
0.8
0.805
z direction from accelometers 2
Acceleration [m/s²]
Time [s]
0.79
0.8
0.81
0.8
0.805
z direction from accelometers 3
Acceleration [m/s²]
Time [s]
0.79
0.8
0.81
0.8
0.805

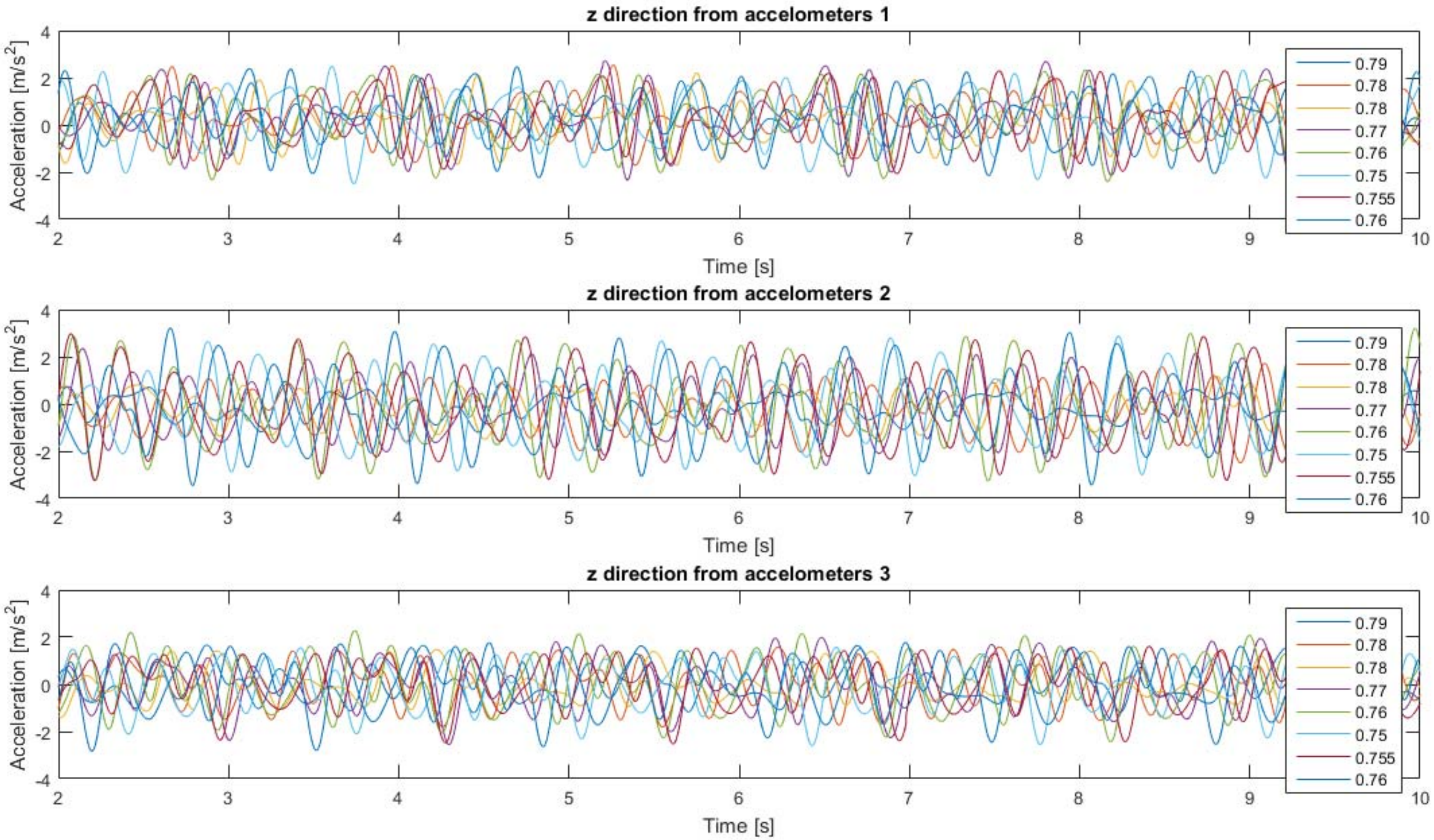
2 - 4 liter


z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.75
0.76
0.77
0.78
0.77
z direction from accelometers 2
Acceleration [m/s^2]
Time [s]
0.75
0.76
0.77
0.78
0.77
z direction from accelometers 3
Acceleration [m/s^2]
Time [s]
0.75
0.76
0.77
0.78
0.77

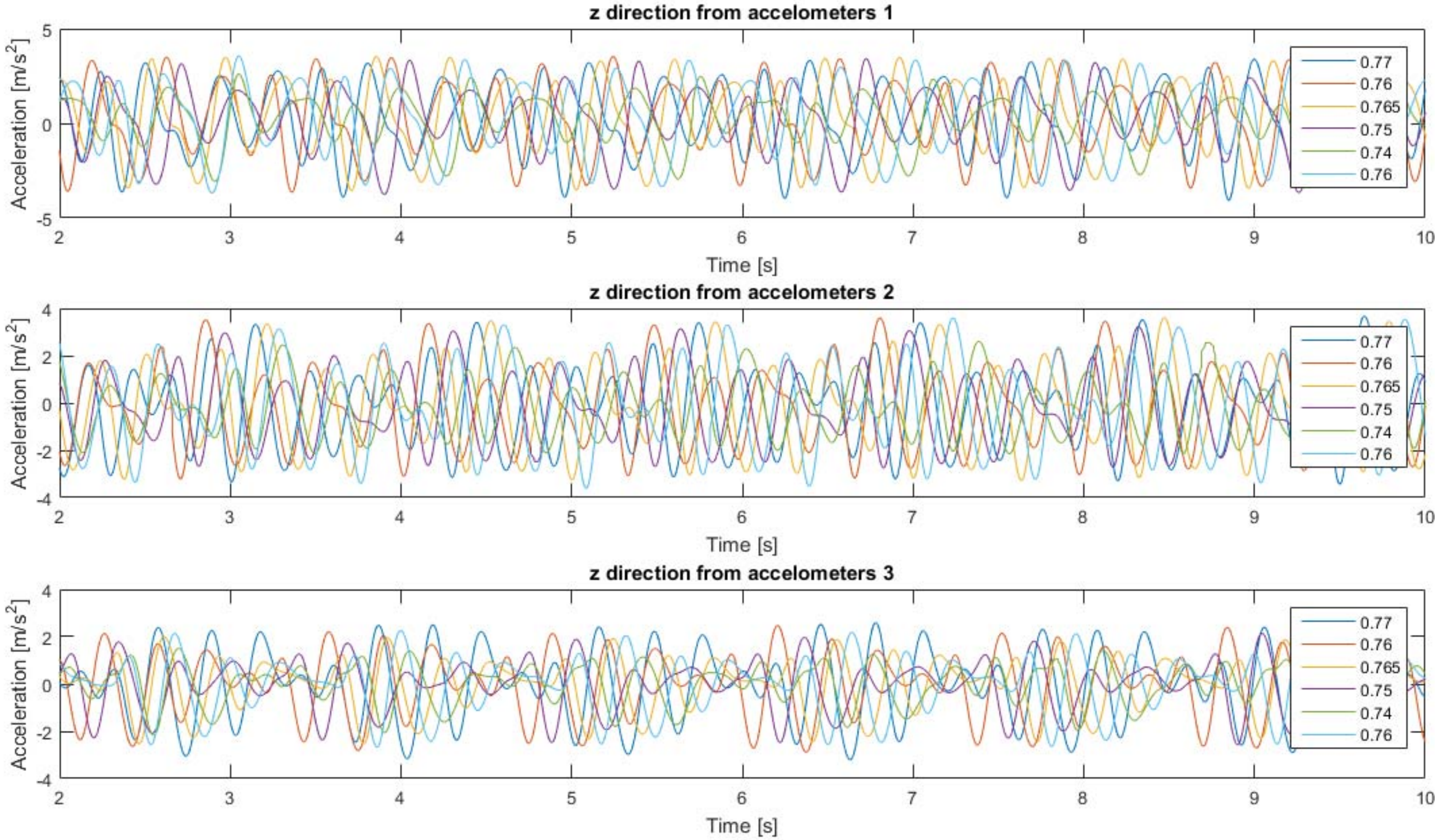
3 - 6 liter


z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.7
0.72
0.74
0.75
0.77
0.78
0.76
z direction from accelometers 2
Acceleration [m/s^2]
time [s]
0.7
0.72
0.74
0.75
0.77
0.78
0.76
z direction from accelometers 3
Acceleration [m/s^2]
time [s]
0.7
0.72
0.74
0.75
0.77
0.78
0.76

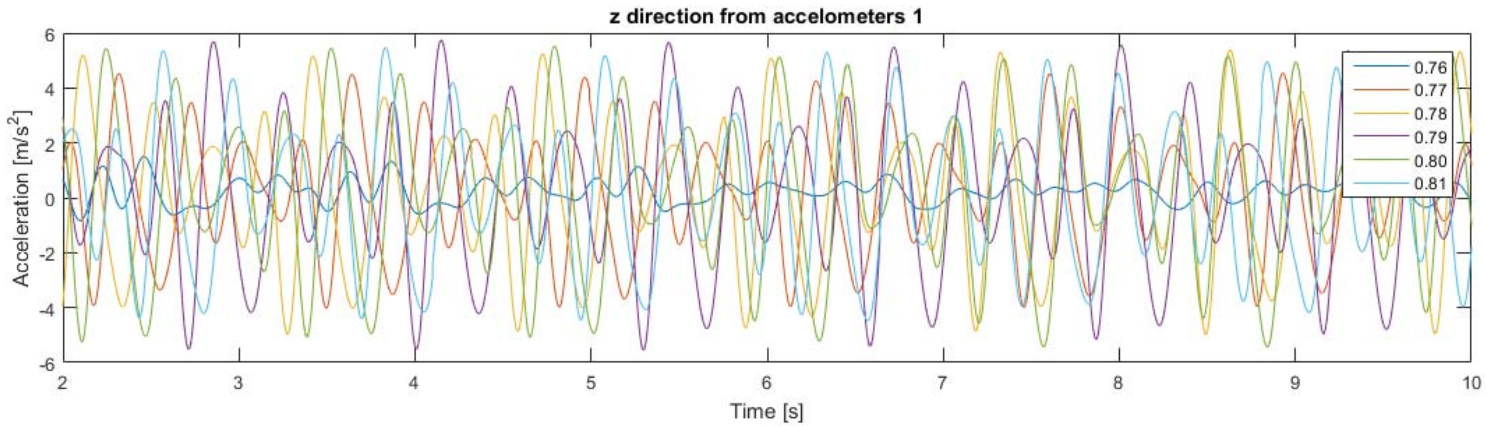
4 - 8 liter

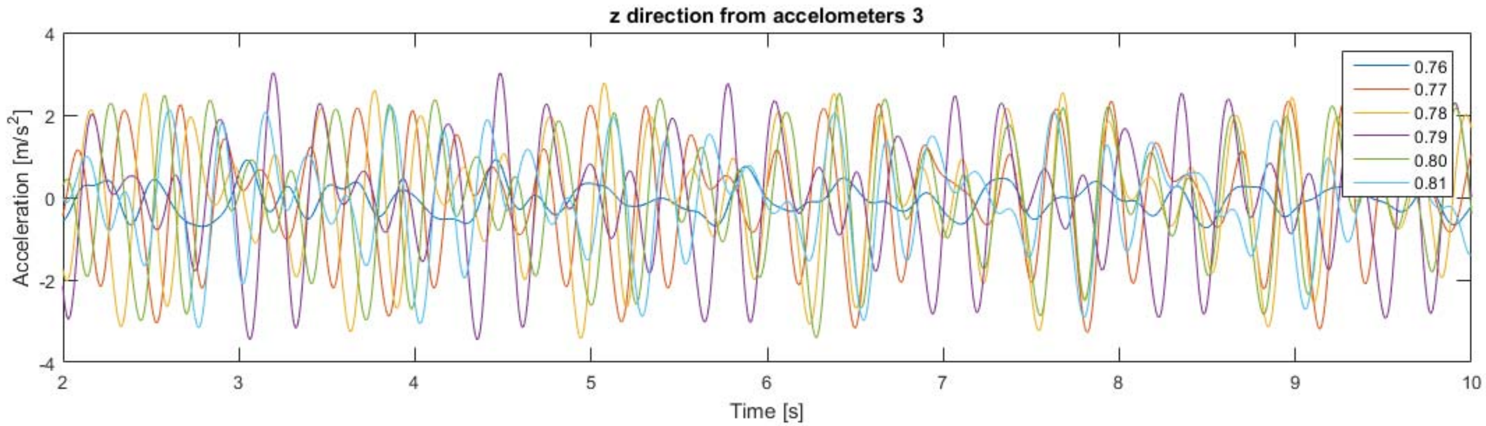

z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.79
0.78
0.78
0.77
0.76
0.75
0.755
0.76
z direction from accelometers 2
Acceleration [m/s^2]
Time [s]
0.79
0.78
0.78
0.77
0.76
0.75
0.755
0.76
z direction from accelometers 3
Acceleration [m/s^2]
Time [s]
0.79
0.78
0.78
0.77
0.76
0.75
0.755
0.76

5 - 10 liter

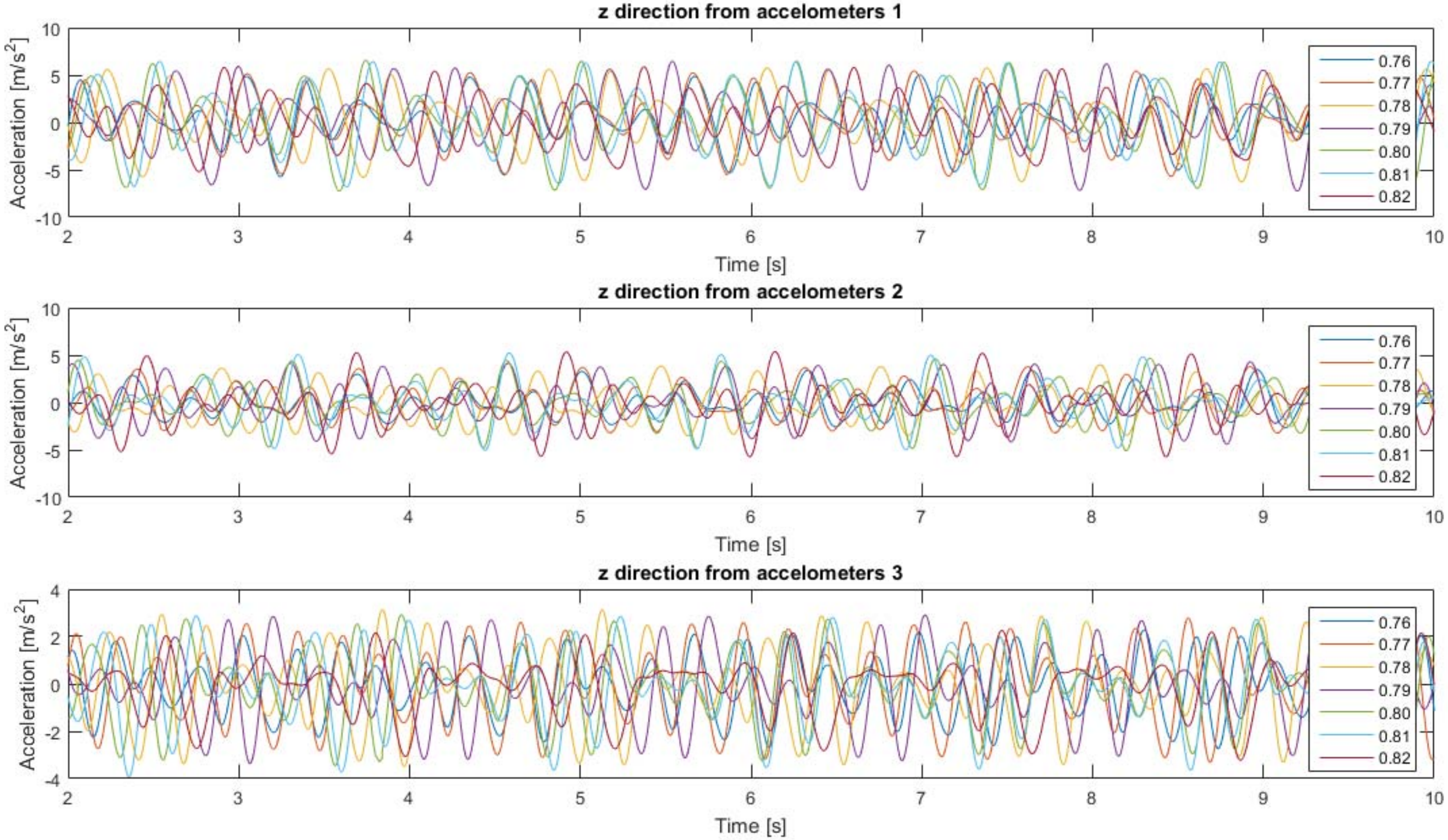

z direction from accelometers 1
Acceleration [m/s²]
Time [s]
0.77
0.76
0.765
0.75
0.74
0.76
z direction from accelometers 2
Acceleration [m/s²]
Time [s]
0.77
0.76
0.765
0.75
0.74
0.76
z direction from accelometers 3
Acceleration [m/s²]
Time [s]
0.77
0.76
0.765
0.75
0.74
0.76

6 - 12 liter

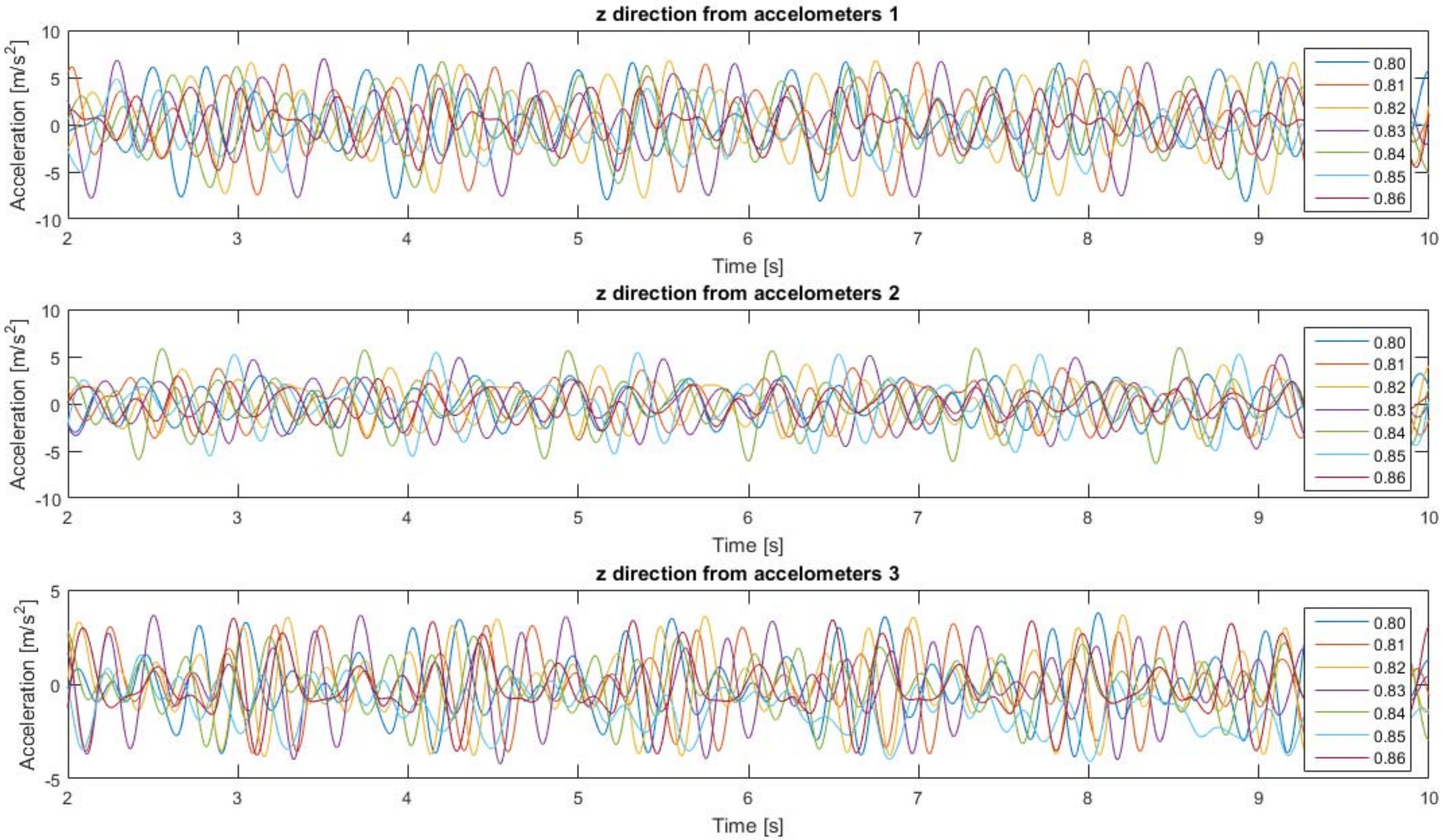

z direction from accelometers 1
Acceleration [m/s²]
Time [s]
0.76
0.77
0.78
0.79
0.80
0.81


z direction from accelometers 3
Acceleration [m/s²]
Time [s]
0.76
0.77
0.78
0.79
0.80
0.81

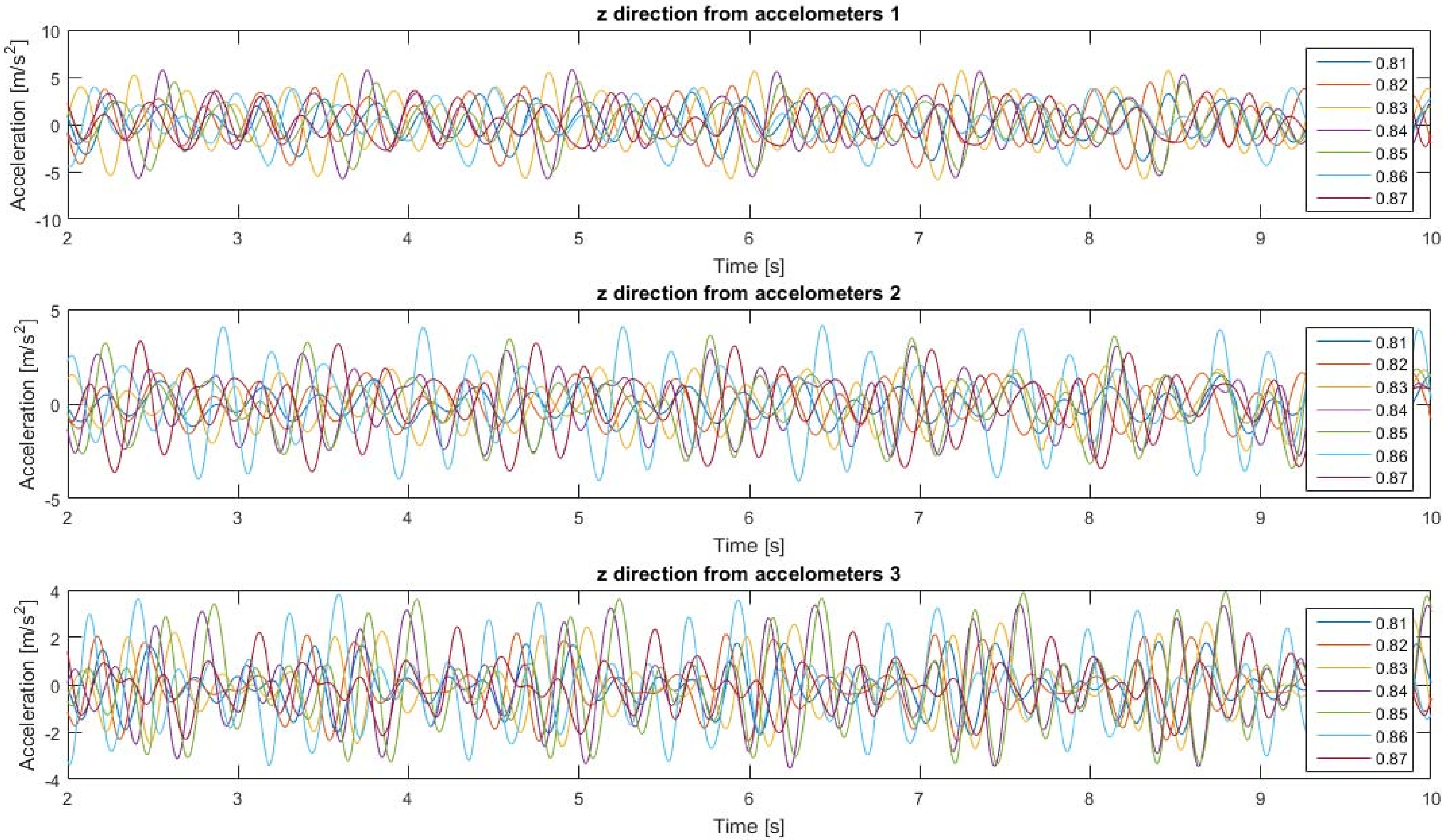
7 - 14 liter


z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
z direction from accelometers 2
Acceleration [m/s^2]
Time [s]
z direction from accelometers 3
Acceleration [m/s^2]
Time [s]
0.76
0.77
0.78
0.79
0.80
0.81
0.82

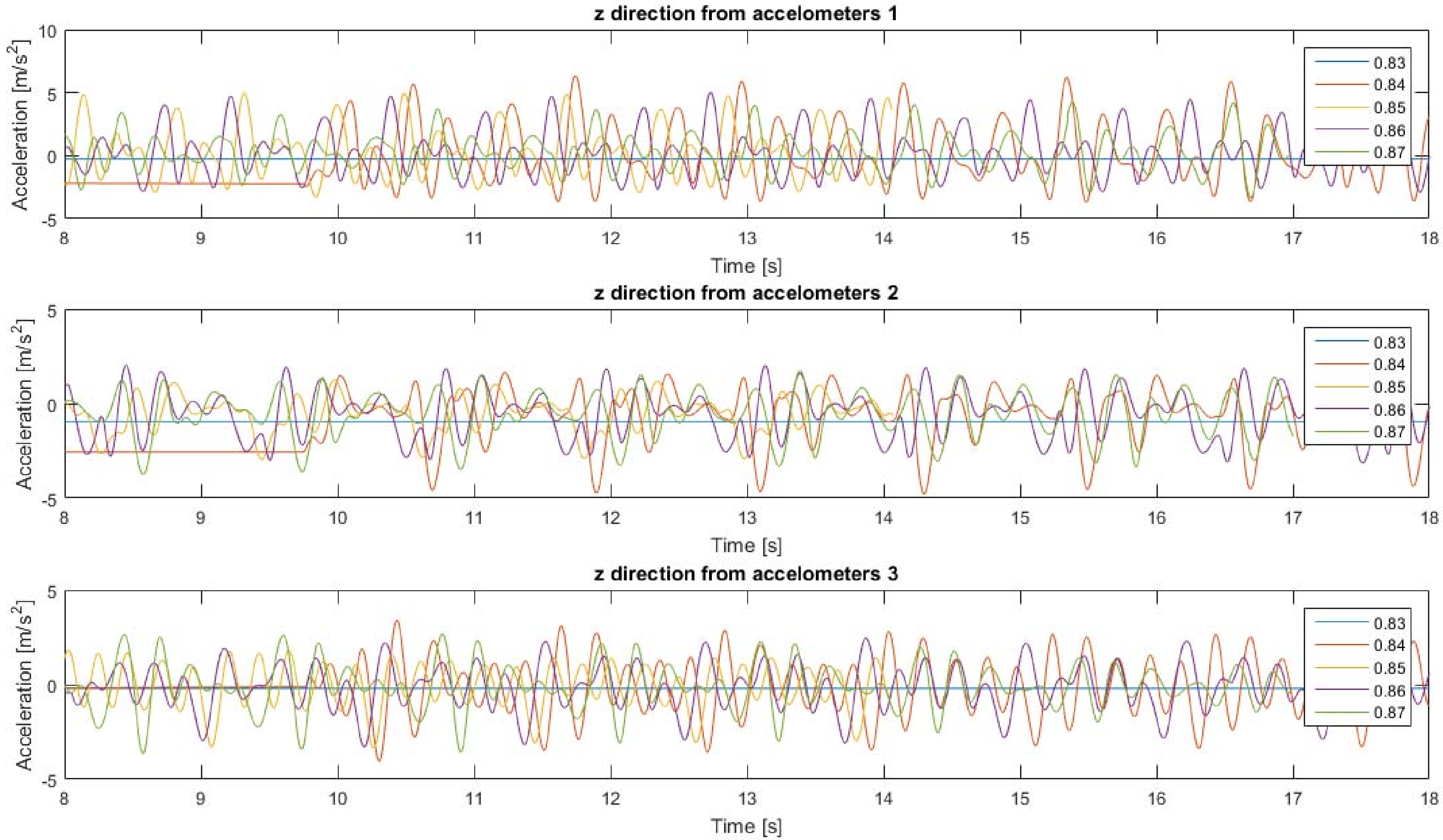
8 - 16 liter


z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.80
0.81
0.82
0.83
0.84
0.85
0.86
z direction from accelometers 2
Acceleration [m/s^2]
Time [s]
z direction from accelometers 3
Acceleration [m/s^2]
Time [s]

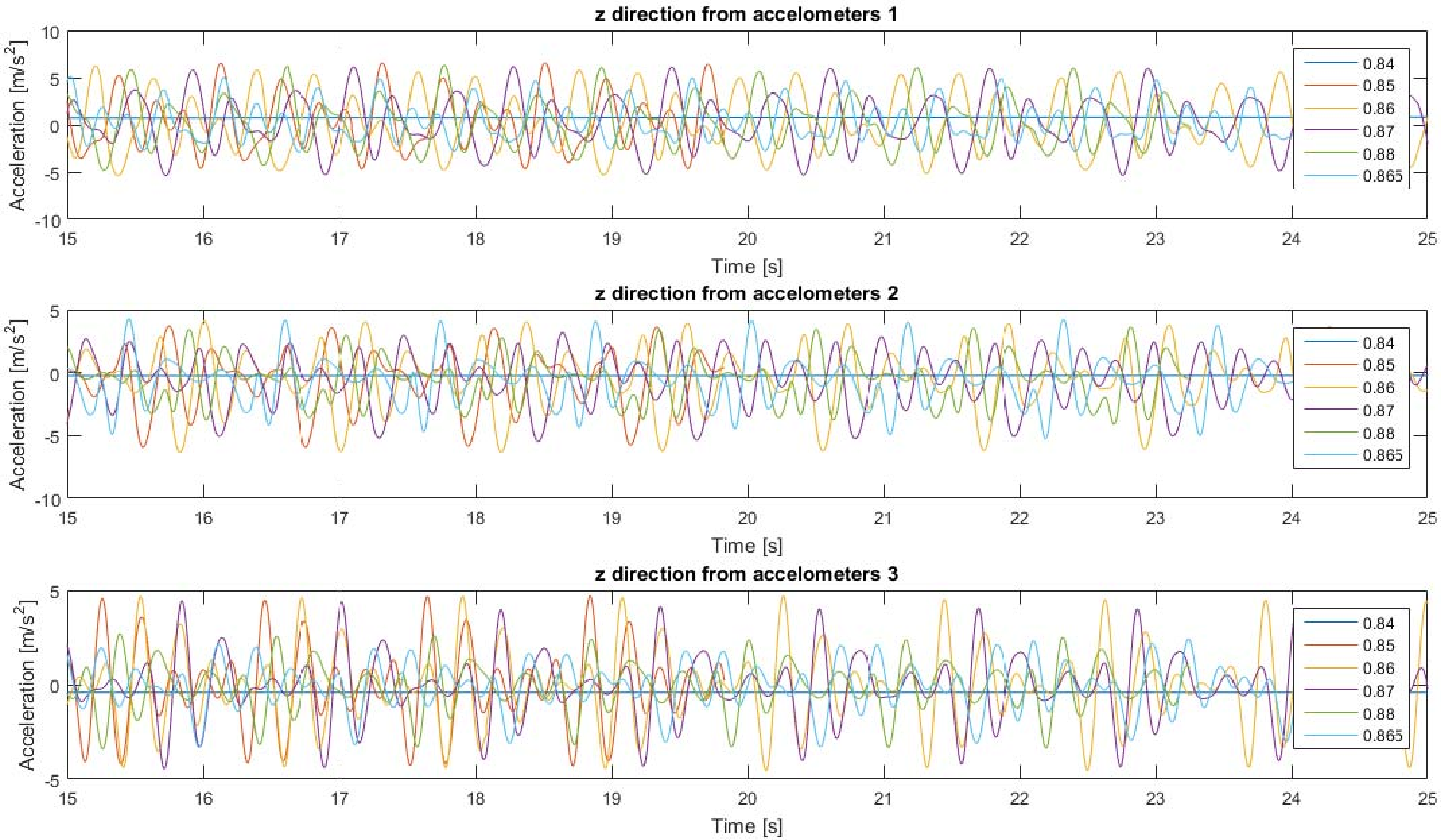
9 - 18 liter


z direction from accelometers 1
Acceleration [m/s²]
Time [s]
0.81
0.82
0.83
0.84
0.85
0.86
0.87
z direction from accelometers 2
Acceleration [m/s²]
Time [s]
0.81
0.82
0.83
0.84
0.85
0.86
0.87
z direction from accelometers 3
Acceleration [m/s²]
Time [s]
0.81
0.82
0.83
0.84
0.85
0.86
0.87

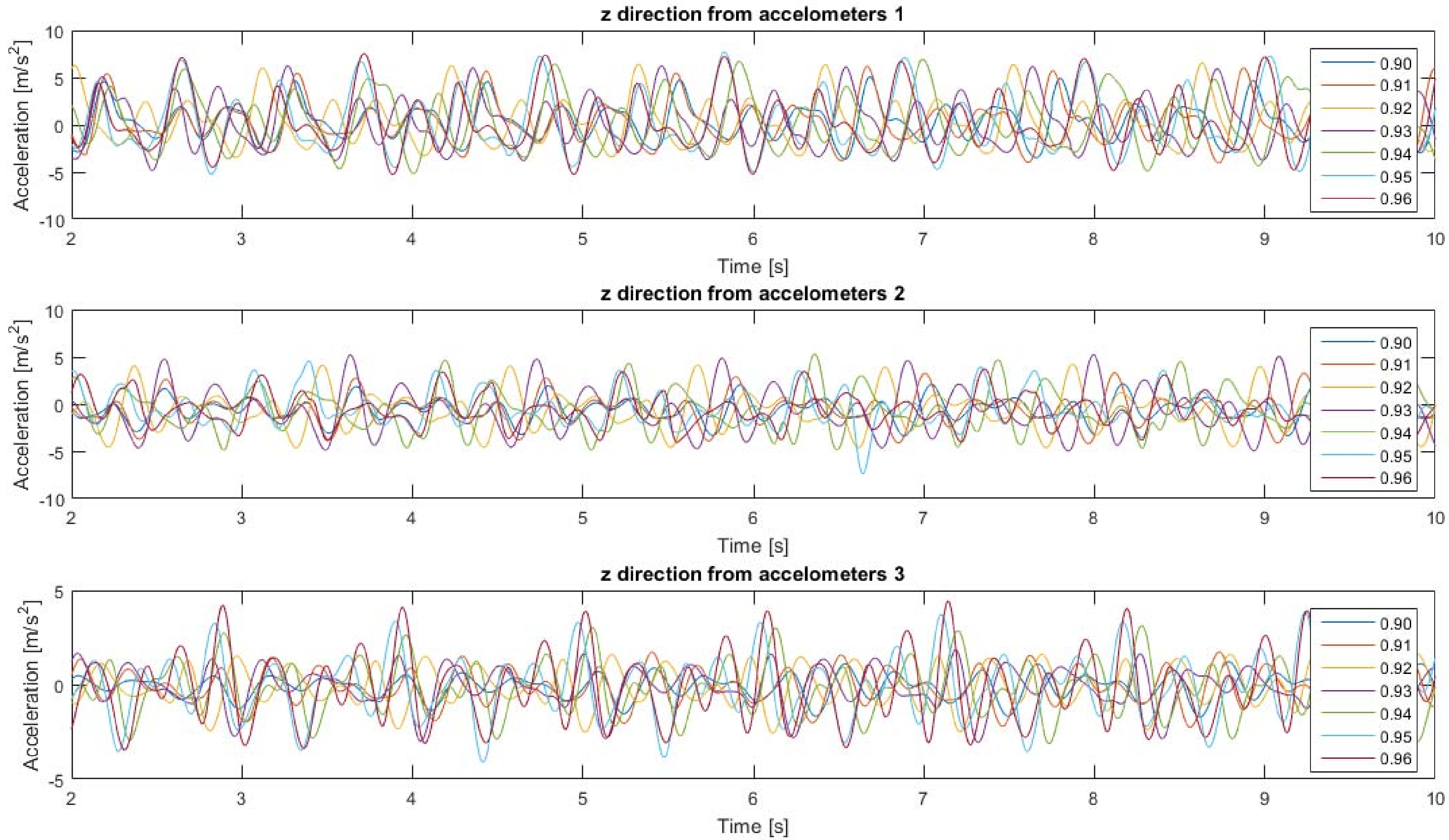
10 - 20 liter


z direction from accelerometers 1
Acceleration [m/s²]
Time [s]
0.83
0.84
0.85
0.86
0.87
z direction from accelerometers 2
Acceleration [m/s²]
Time [s]
0.83
0.84
0.85
0.86
0.87
z direction from accelerometers 3
Acceleration [m/s²]
Time [s]
0.83
0.84
0.85
0.86
0.87

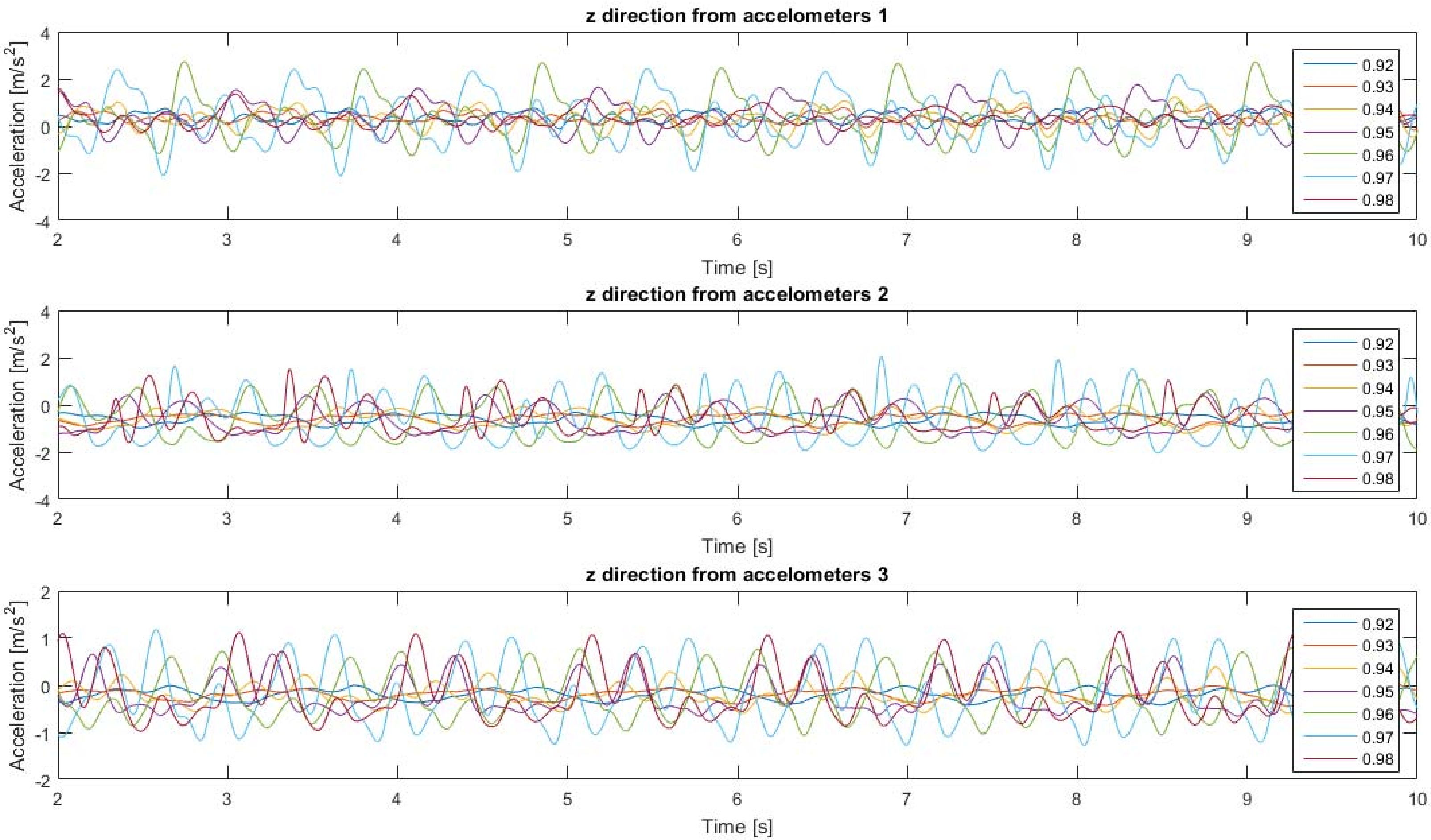
11 - 22 liter


z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.84
0.85
0.86
0.87
0.88
0.865
z direction from accelometers 2
Acceleration [m/s^2]
Time [s]
z direction from accelometers 3
Acceleration [m/s^2]
Time [s]

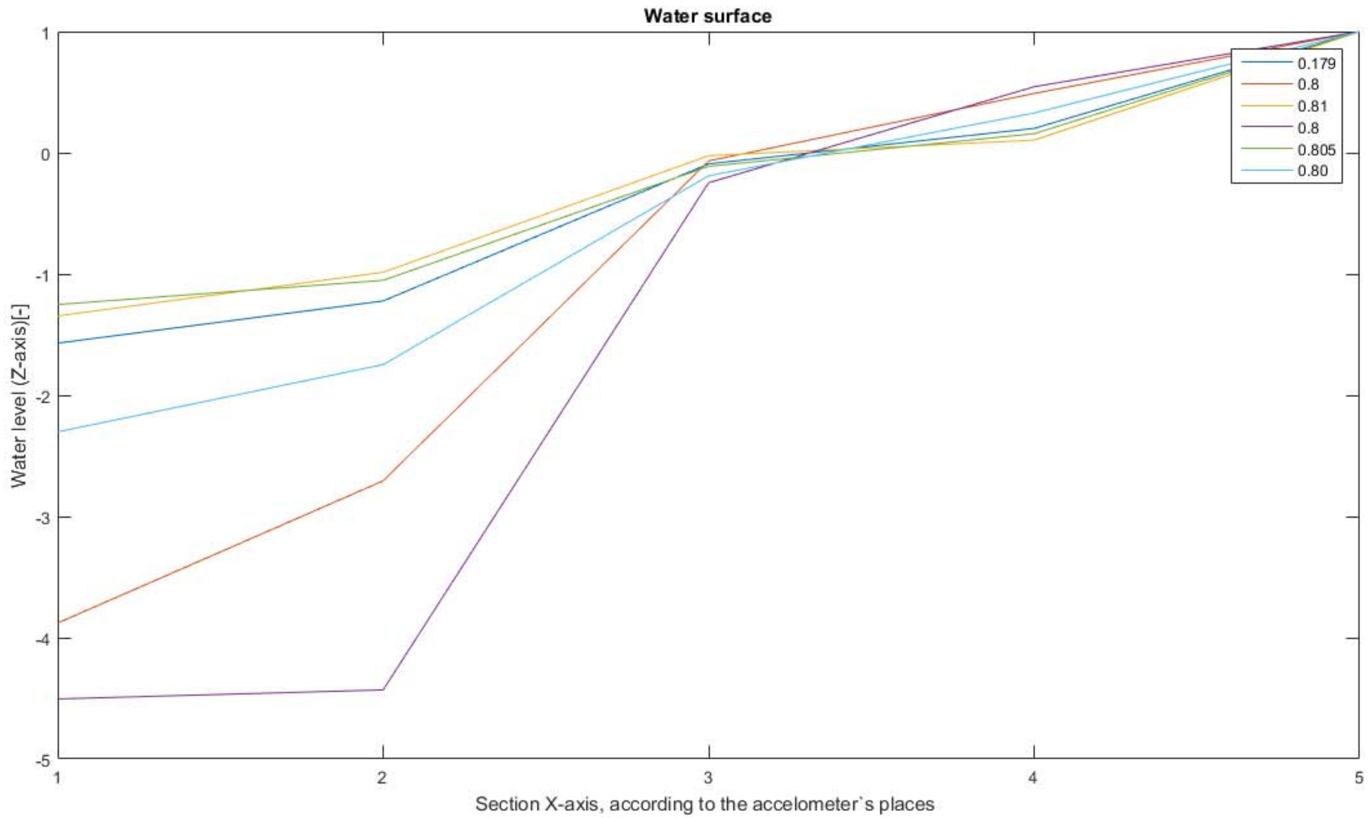
12 - 24 liter


z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.90
0.91
0.92
0.93
0.94
0.95
0.96
z direction from accelometers 2
Acceleration [m/s^2]
Time [s]
0.90
0.91
0.92
0.93
0.94
0.95
0.96
z direction from accelometers 3
Acceleration [m/s^2]
Time [s]
0.90
0.91
0.92
0.93
0.94
0.95
0.96

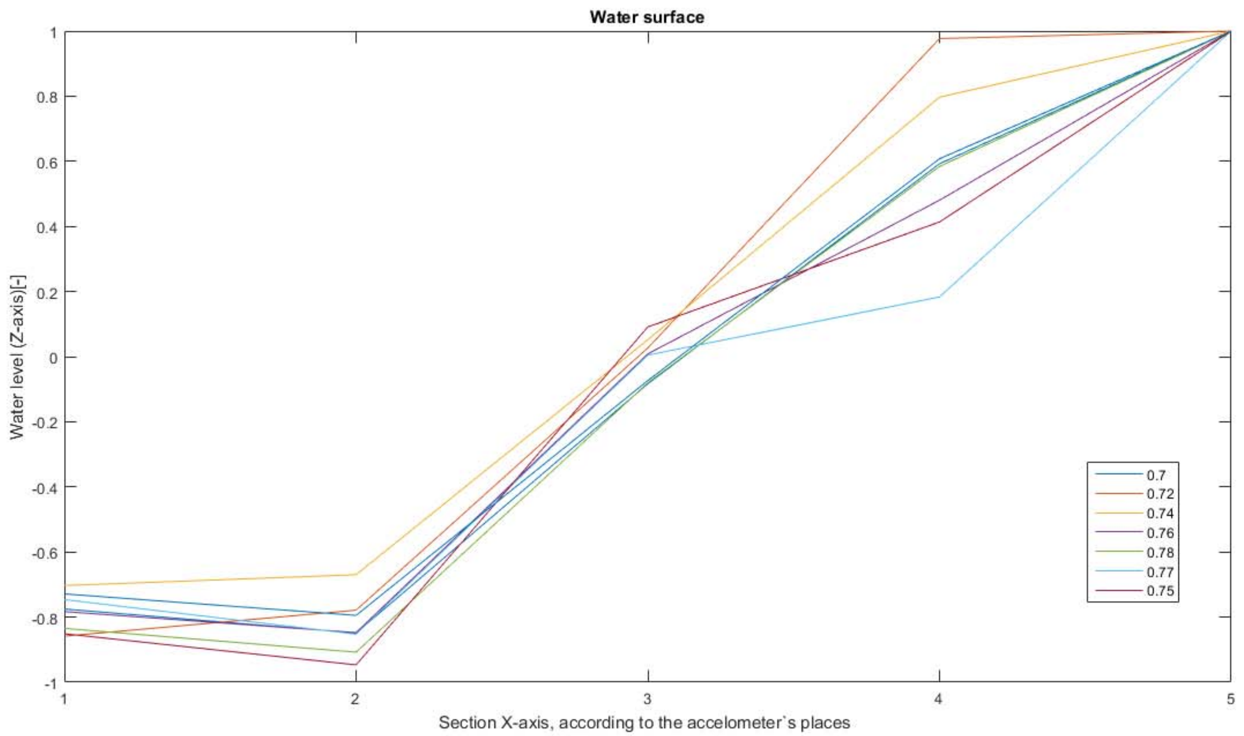
13 - 30 liter


z direction from accelometers 1
Acceleration [m/s^2]
Time [s]
0.92
0.93
0.94
0.95
0.96
0.97
0.98
z direction from accelometers 2
Acceleration [m/s^2]
Time [s]
0.92
0.93
0.94
0.95
0.96
0.97
0.98
z direction from accelometers 3
Acceleration [m/s^2]
Time [s]
0.92
0.93
0.94
0.95
0.96
0.97
0.98

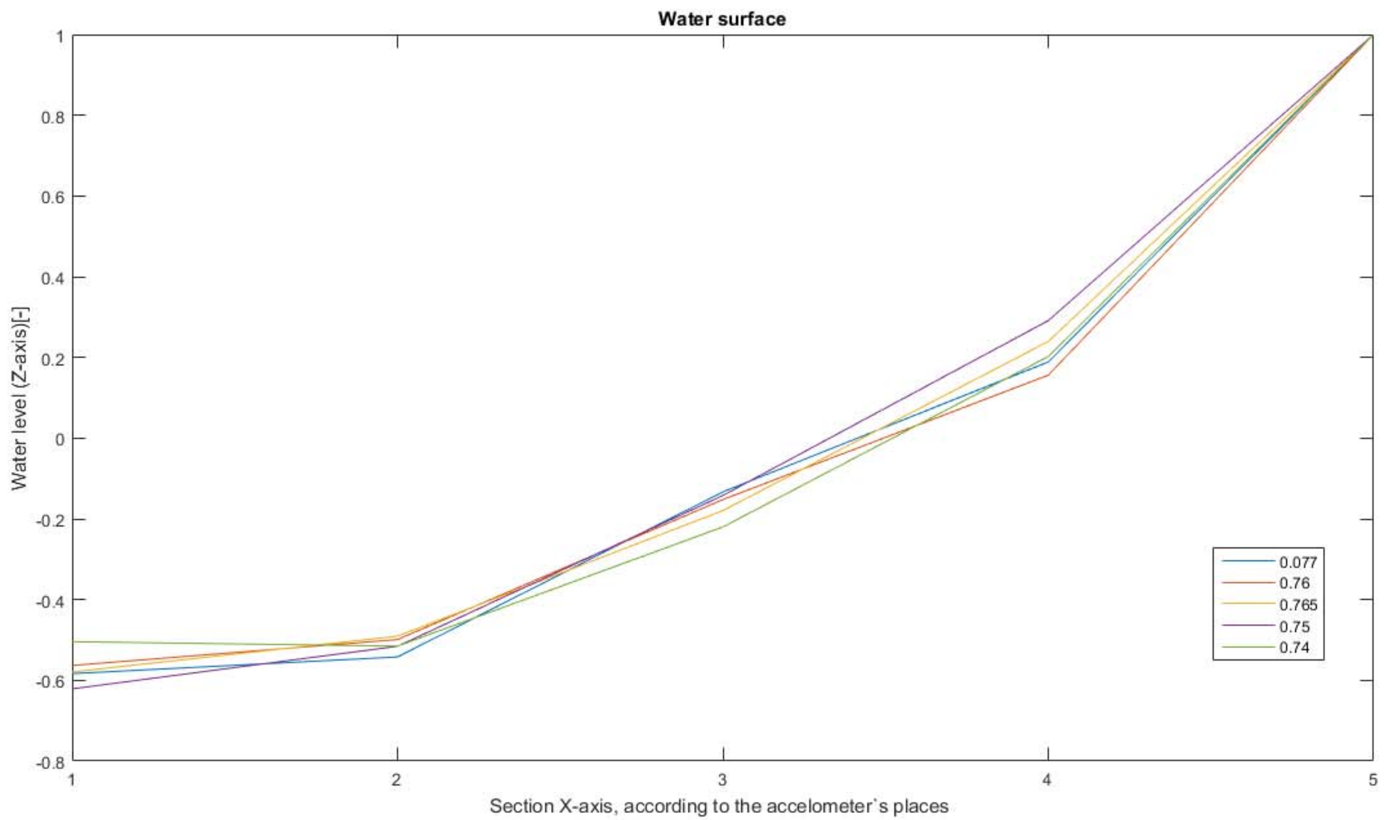
14 - 34 liter


Water surface
Water level (Z-axis)[-]
Section X-axis, according to the accelometer`s places
0.179
0.8
0.81
0.8
0.805
0.80
1
0
-1
-2
-3
-4
-5
1
2
3
4
5

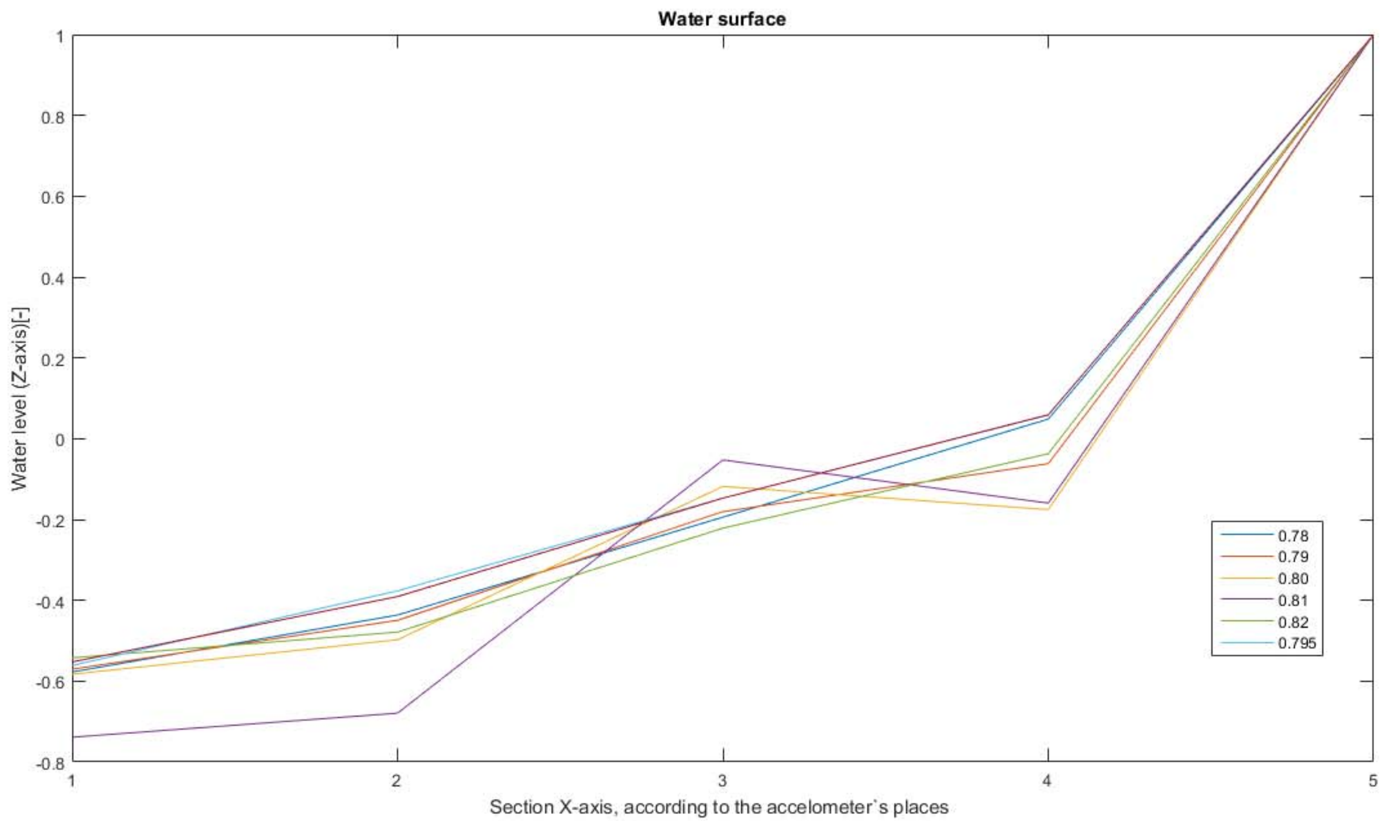
4 liter


Water surface
Water level (Z-axis)[-]
Section X-axis, according to the accelometer`s places
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
1
2
3
4
5
0.7
0.72
0.74
0.76
0.78
0.77
0.75

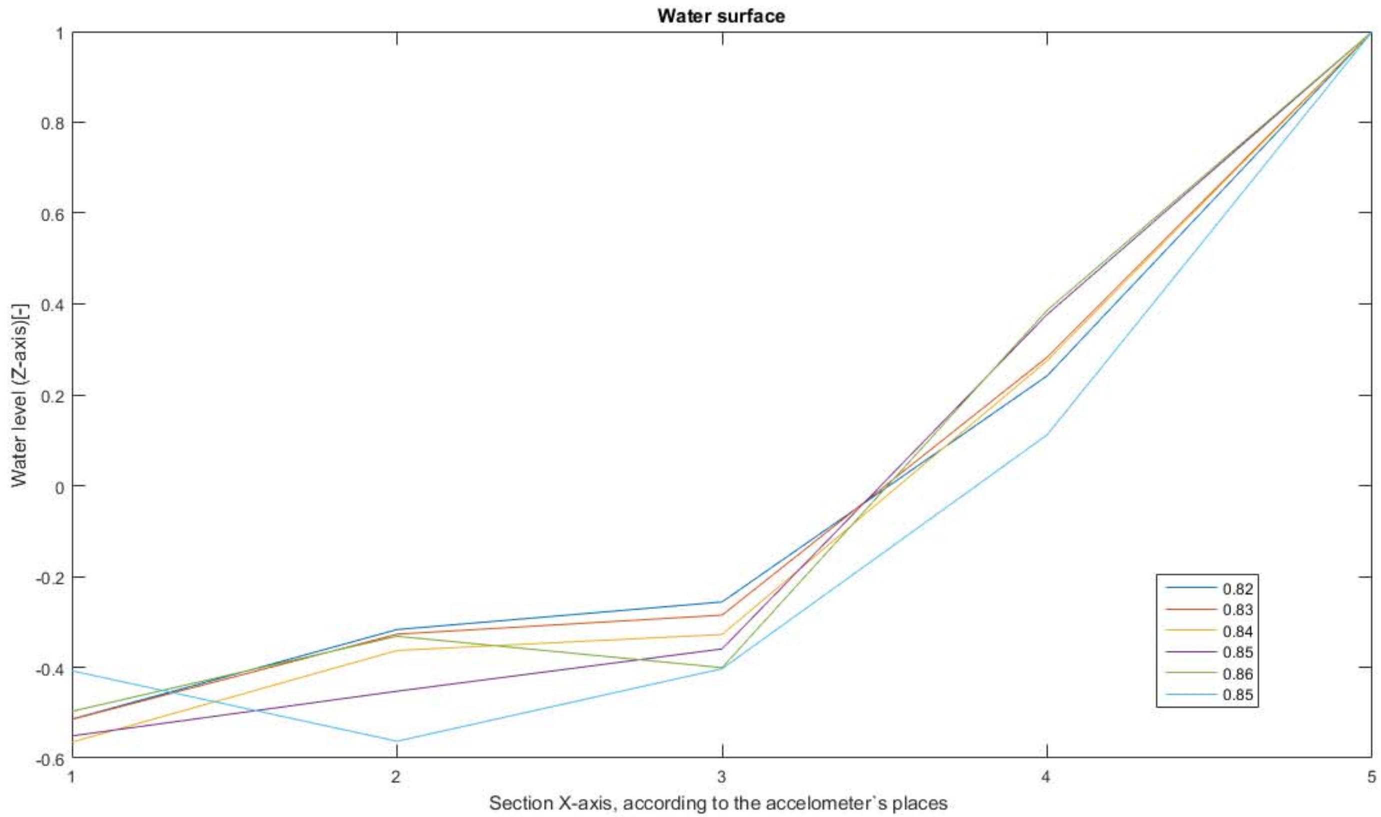
8 liter


Water surface
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Water level (Z-axis)[-]
1
2
3
4
5
Section X-axis, according to the accelometer`s places
0.077
0.76
0.765
0.75
0.74

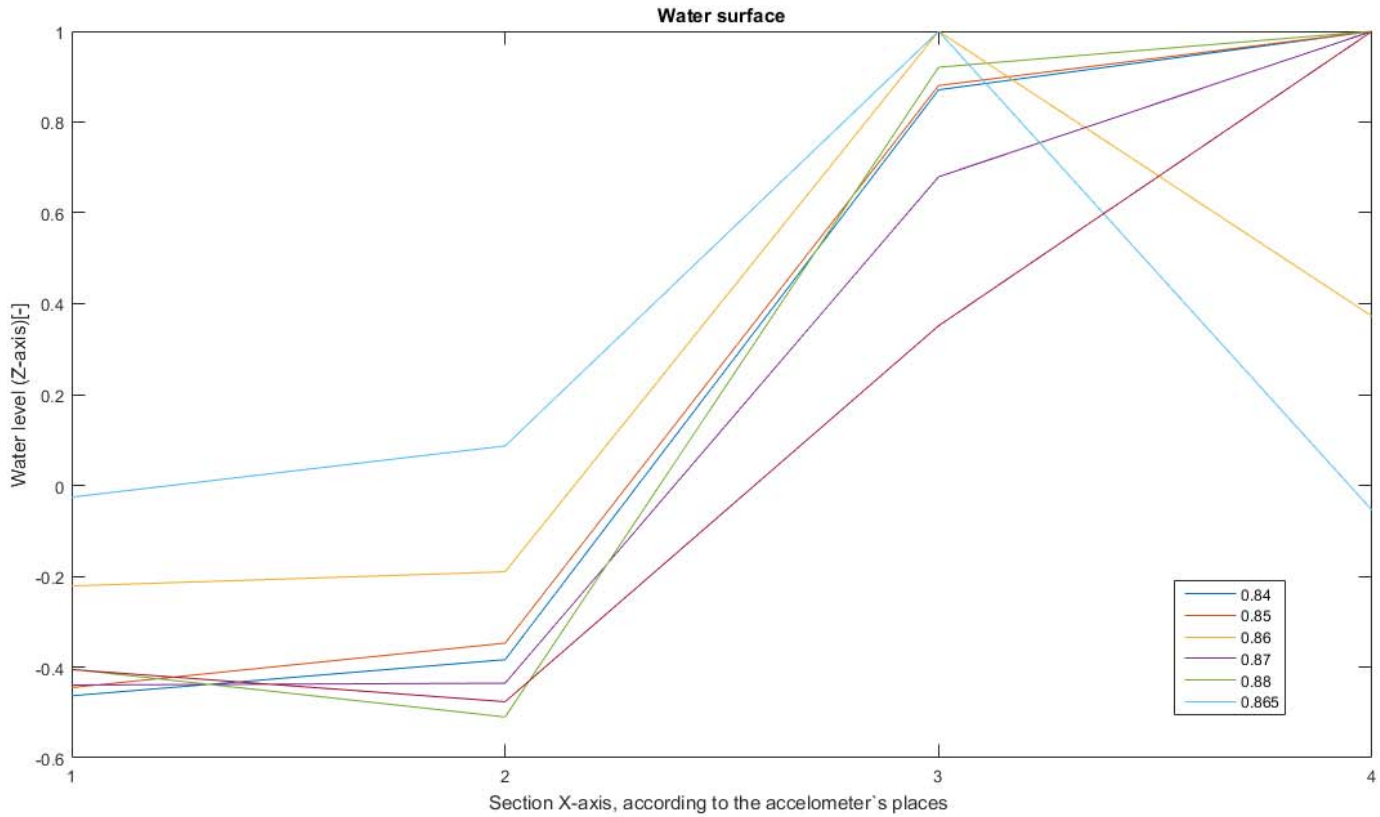
12 liter


Water surface
Water level (Z-axis)[-]
Section X-axis, according to the accelometer`s places
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1
2
3
4
5
0.78
0.79
0.80
0.81
0.82
0.795

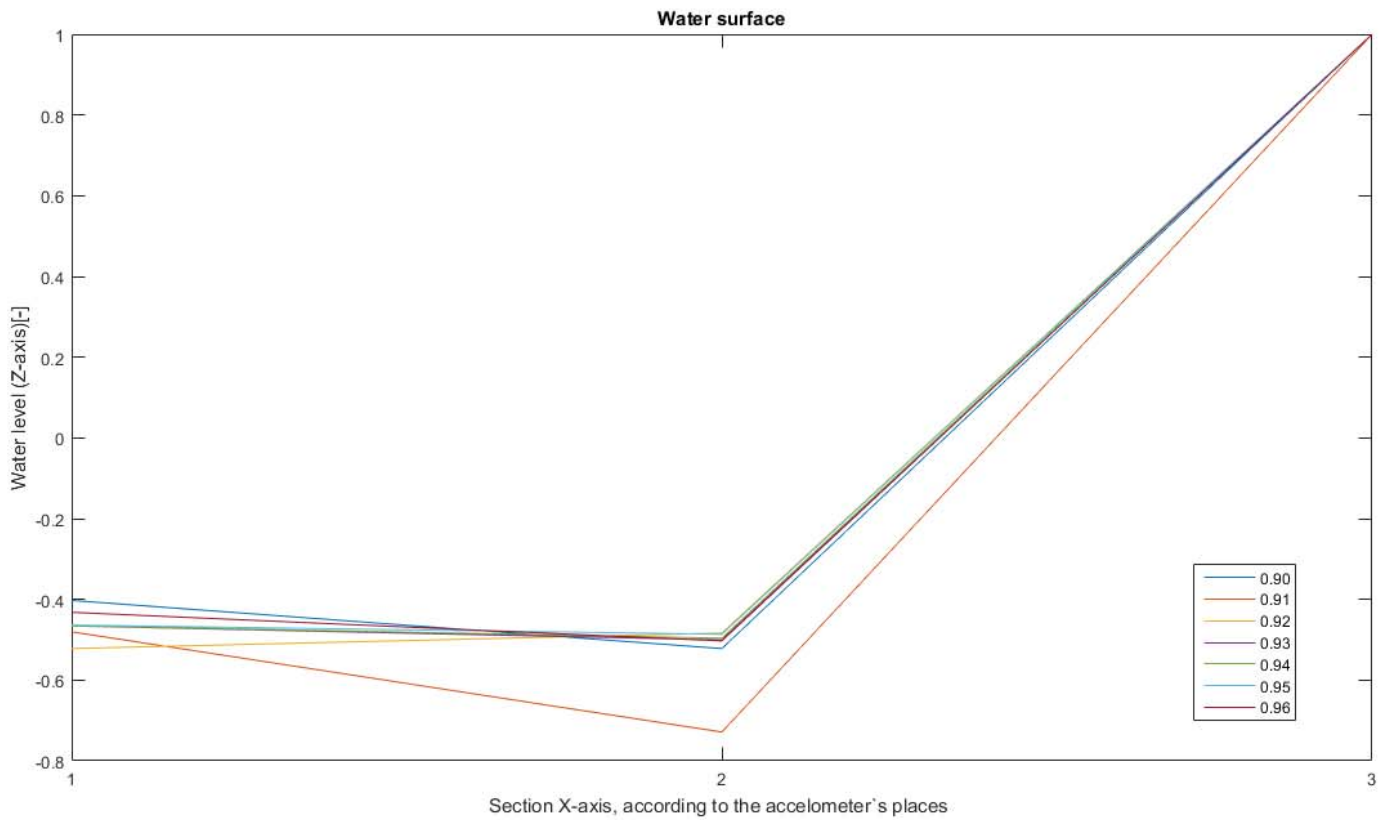
16 liter


Water surface
Water level (Z-axis)[-]
Section X-axis, according to the accelometer`s places
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1
2
3
4
5
0.82
0.83
0.84
0.85
0.86
0.85

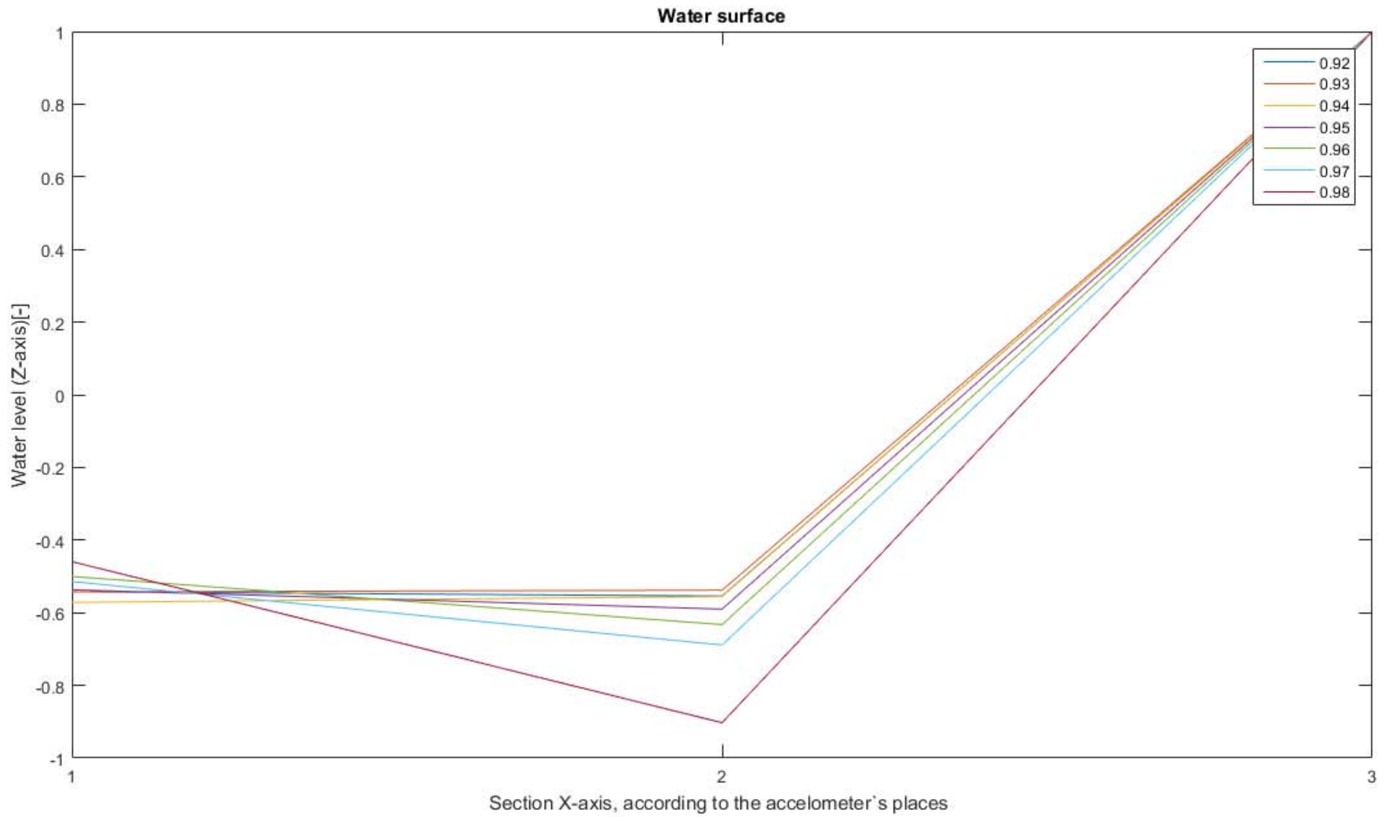
20 liter


Water surface
Water level (Z-axis)[-]
Section X-axis, according to the accelometer`s places
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1
2
3
4
0.84
0.85
0.86
0.87
0.88
0.865

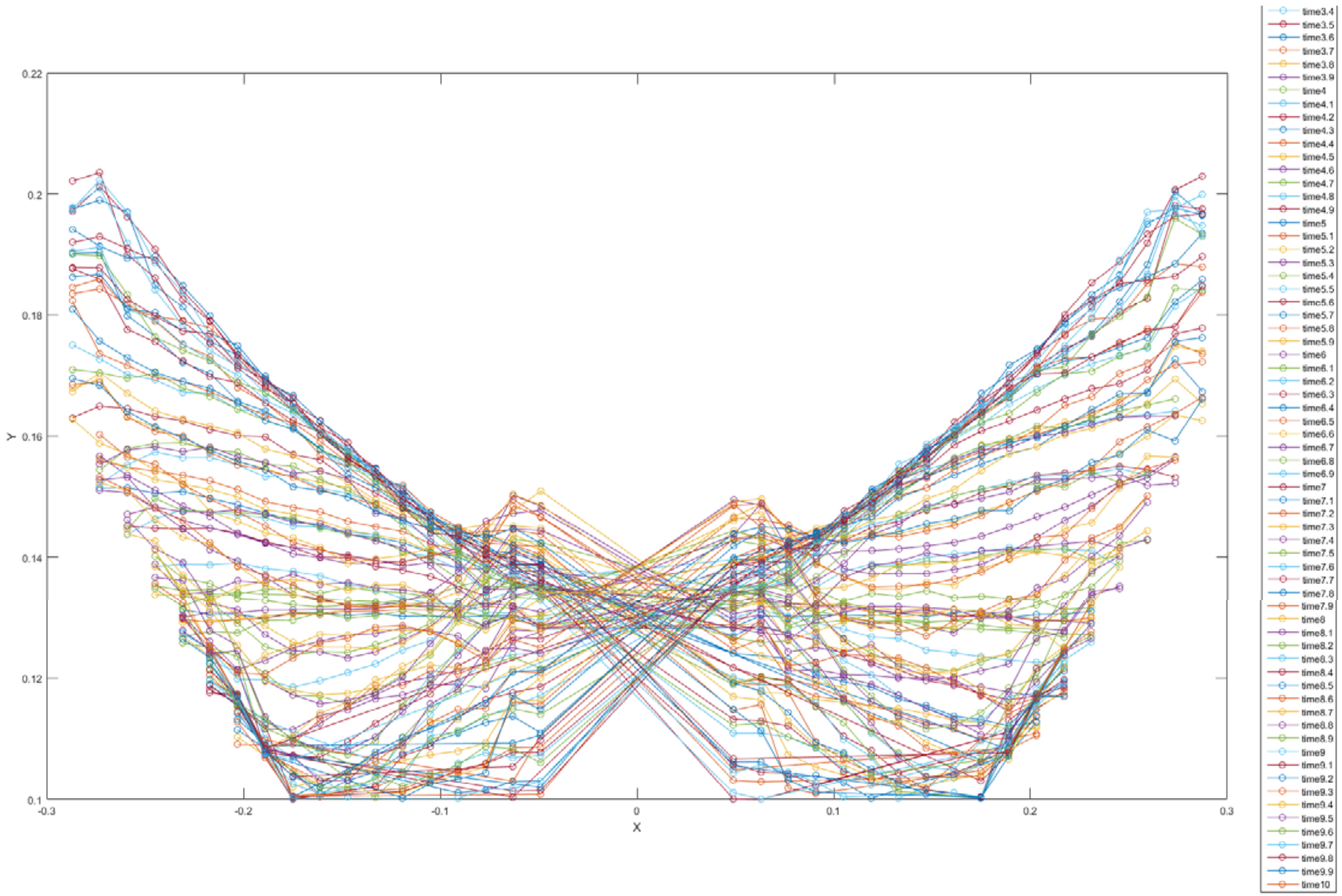
24 liter


Water surface
Water level (Z-axis)[-]
Section X-axis, according to the accelometer`s places
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1
2
3
0.90
0.91
0.92
0.93
0.94
0.95
0.96

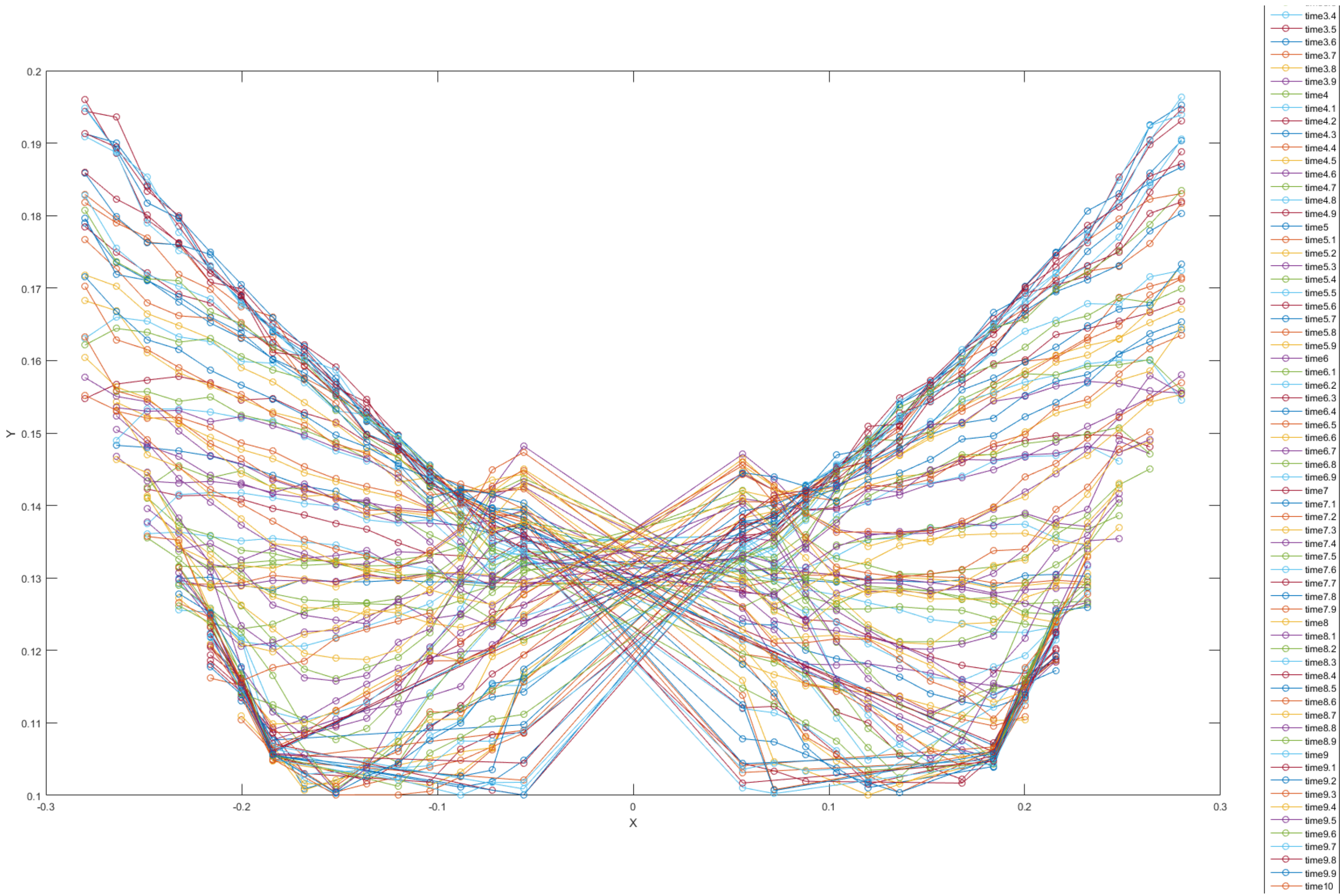
30 liter


Water surface
Water level (Z-axis)[-]
Section X-axis, according to the accelometer`s places
0.92
0.93
0.94
0.95
0.96
0.97
0.98

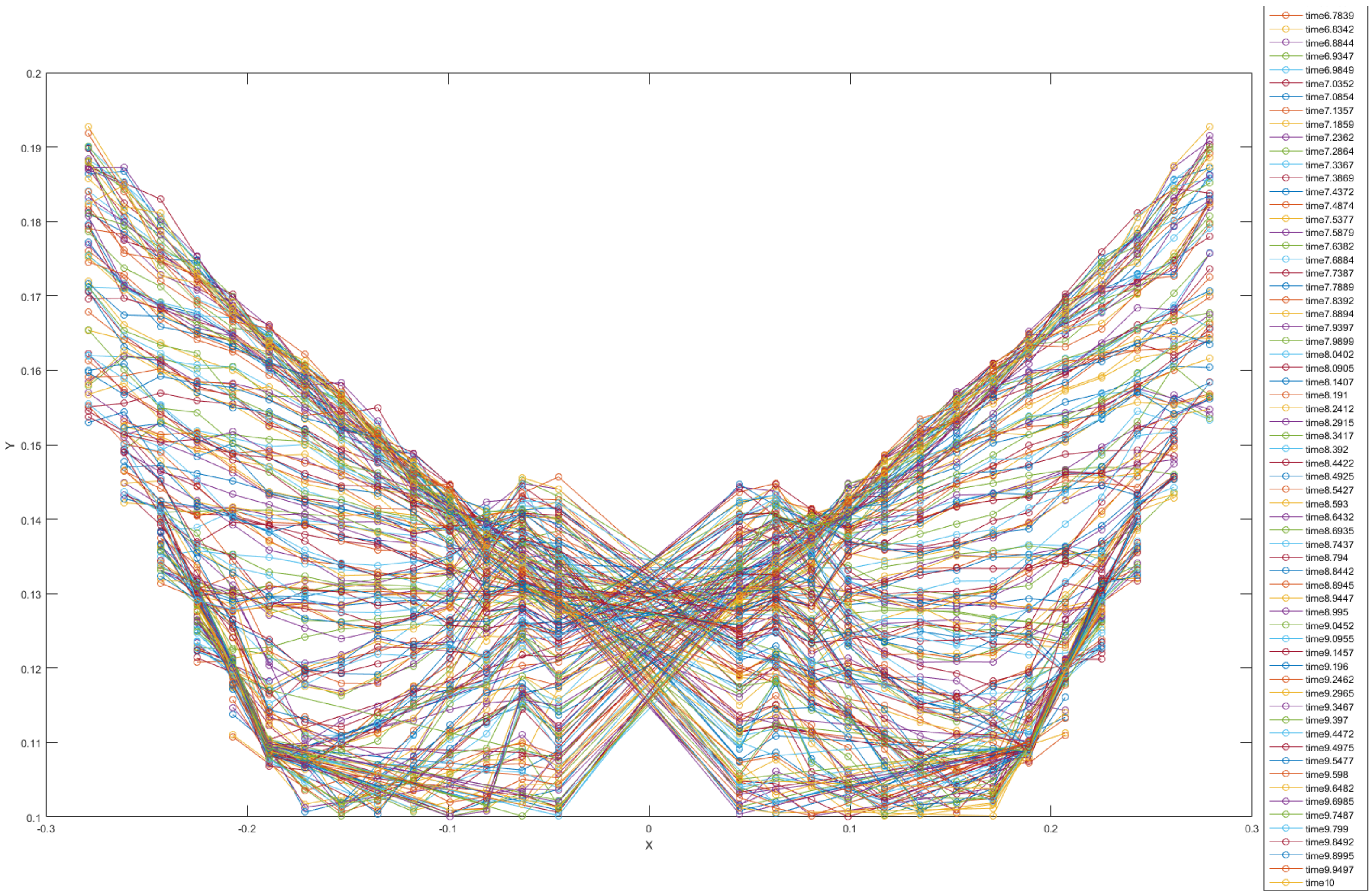
34 liter


0.22
0.2
0.18
0.16
0.14
0.12
0.1
Y
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
X
time3.4
time3.5
time3.6
time3.7
time3.8
time3.9
time4
time4.1
time4.2
time4.3
time4.4
time4.5
time4.6
time4.7
time4.8
time4.9
time5
time5.1
time5.2
time5.3
time5.4
time5.5
time5.6
time5.7
time5.8
time5.9
time6
time6.1
time6.2
time6.3
time6.4
time6.5
time6.6
time6.7
time6.8
time6.9
time7
time7.1
time7.2
time7.3
time7.4
time7.5
time7.6
time7.7
time7.8
time7.9
time8
time8.1
time8.2
time8.3
time8.4
time8.5
time8.6
time8.7
time8.8
time8.9
time9
time9.1
time9.2
time9.3
time9.4
time9.5
time9.6
time9.7
time9.8
time9.9
time10

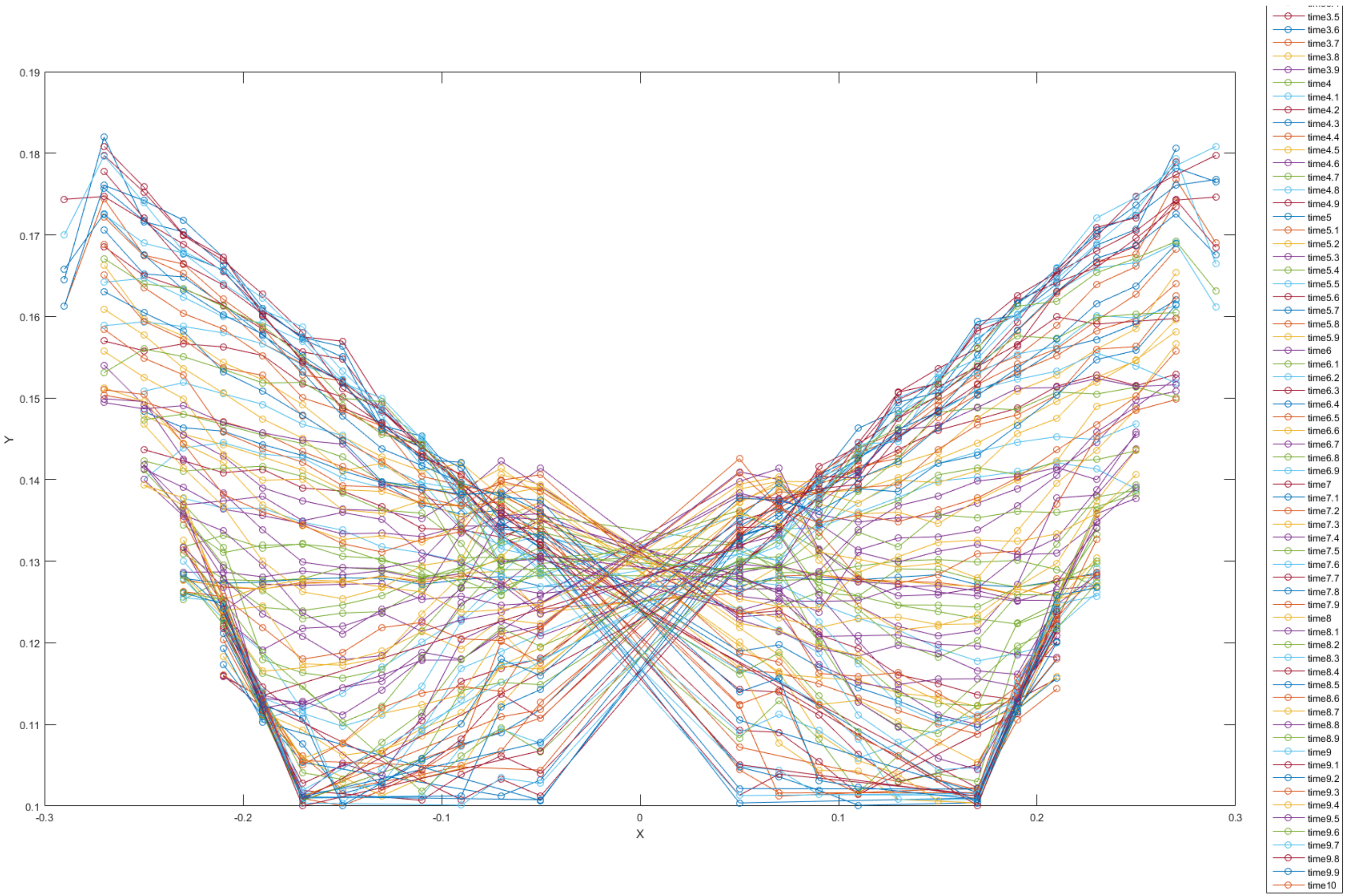
0.74 Hz – Sr = 14


0.2
0.19
0.18
0.17
0.16
Y 0.15
0.14
0.13
0.12
0.11
0.1
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
X
time3.4
time3.5
time3.6
time3.7
time3.8
time3.9
time4
time4.1
time4.2
time4.3
time4.4
time4.5
time4.6
time4.7
time4.8
time4.9
time5
time5.1
time5.2
time5.3
time5.4
time5.5
time5.6
time5.7
time5.8
time5.9
time6
time6.1
time6.2
time6.3
time6.4
time6.5
time6.6
time6.7
time6.8
time6.9
time7
time7.1
time7.2
time7.3
time7.4
time7.5
time7.6
time7.7
time7.8
time7.9
time8
time8.1
time8.2
time8.3
time8.4
time8.5
time8.6
time8.7
time8.8
time8.9
time9
time9.1
time9.2
time9.3
time9.4
time9.5
time9.6
time9.7
time9.8
time9.9
time10

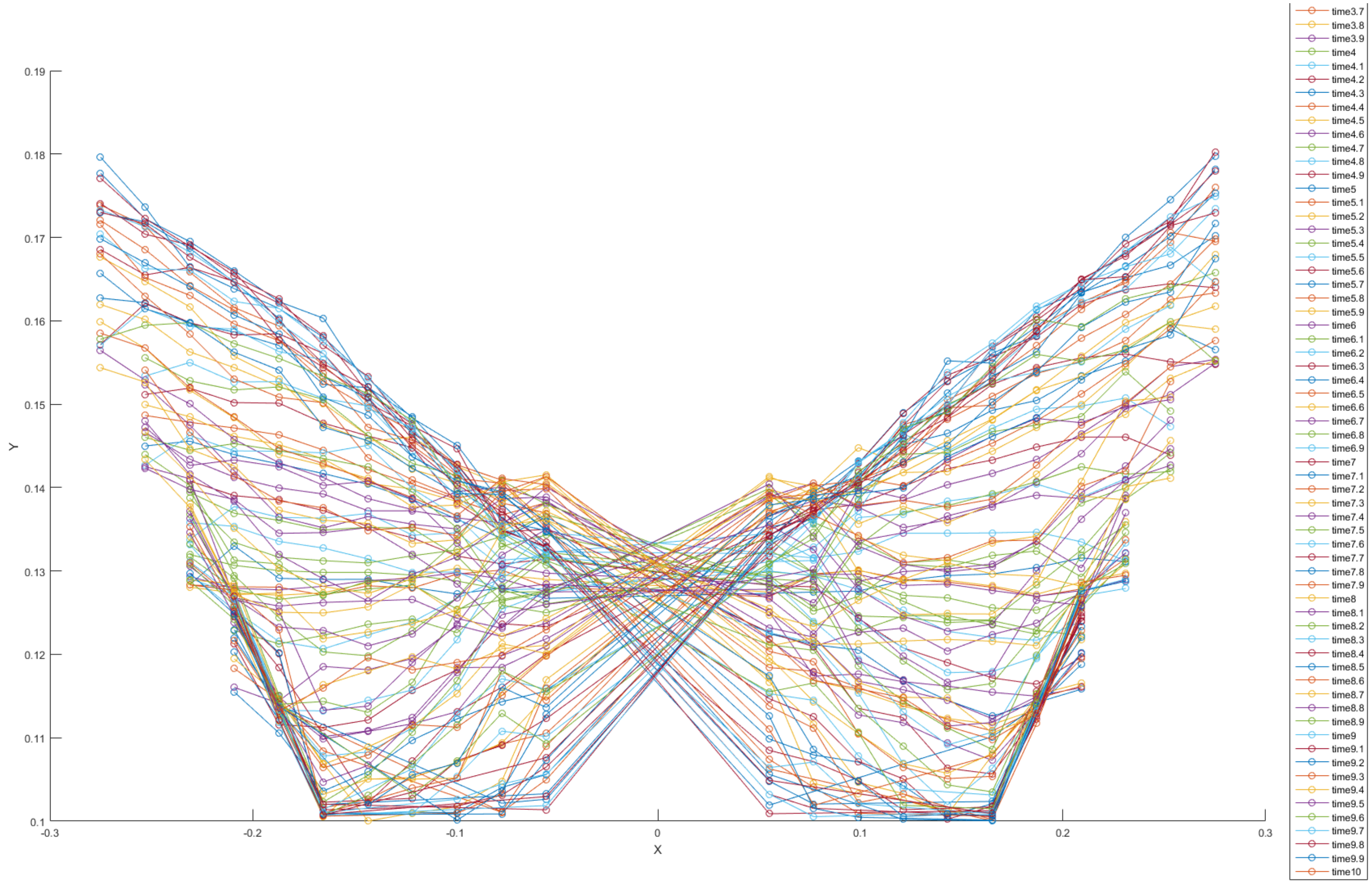
0.74 Hz – Sr = 16


Y
X
0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
time6.7839
time6.8342
time6.8844
time6.9347
time6.9849
time7.0352
time7.0854
time7.1357
time7.1859
time7.2362
time7.2864
time7.3367
time7.3869
time7.4372
time7.4874
time7.5377
time7.5879
time7.6382
time7.6884
time7.7387
time7.7889
time7.8392
time7.8894
time7.9397
time7.9899
time8.0402
time8.0905
time8.1407
time8.191
time8.2412
time8.2915
time8.3417
time8.392
time8.4422
time8.4925
time8.5427
time8.593
time8.6432
time8.6935
time8.7437
time8.794
time8.8442
time8.8945
time8.9447
time8.995
time9.0452
time9.0955
time9.1457
time9.196
time9.2462
time9.2965
time9.3467
time9.397
time9.4472
time9.4975
time9.5477
time9.598
time9.6482
time9.6985
time9.7487
time9.799
time9.8492
time9.8995
time9.9497
time10

0.74 Hz – Sr = 18


0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
Y
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
X
time3.5
time3.6
time3.7
time3.8
time3.9
time4
time4.1
time4.2
time4.3
time4.4
time4.5
time4.6
time4.7
time4.8
time4.9
time5
time5.1
time5.2
time5.3
time5.4
time5.5
time5.6
time5.7
time5.8
time5.9
time6
time6.1
time6.2
time6.3
time6.4
time6.5
time6.6
time6.7
time6.8
time6.9
time7
time7.1
time7.2
time7.3
time7.4
time7.5
time7.6
time7.7
time7.8
time7.9
time8
time8.1
time8.2
time8.3
time8.4
time8.5
time8.6
time8.7
time8.8
time8.9
time9
time9.1
time9.2
time9.3
time9.4
time9.5
time9.6
time9.7
time9.8
time9.9
time10

0.74 Hz – Sr = 20


0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
Y
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
X
time3.7
time3.8
time3.9
time4
time4.1
time4.2
time4.3
time4.4
time4.5
time4.6
time4.7
time4.8
time4.9
time5
time5.1
time5.2
time5.3
time5.4
time5.5
time5.6
time5.7
time5.8
time5.9
time6
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time6.4
time6.5
time6.6
time6.7
time6.8
time6.9
time7
time7.1
time7.2
time7.3
time7.4
time7.5
time7.6
time7.7
time7.8
time7.9
time8
time8.1
time8.2
time8.3
time8.4
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time8.6
time8.7
time8.8
time8.9
time9
time9.1
time9.2
time9.3
time9.4
time9.5
time9.6
time9.7
time9.8
time9.9
time10

0.74 Hz – Sr = 22